Entre Ciencia e Ingeniería, ISSN 1909-8367
Año 7. No. 14 - Segundo Semestre de 2013, página 24 - 31

Solutions for the fluids between parallel and porous walls[1]

Soluciones para los fluidos entre paredes paralelas y porosas

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Recibido Octubre 16 de 2012 - Aceptado Noviembre 15 de 2013

Abstract **- A conducting fluid is continuously injected or ejected through a pair of parallel porous walls and it escapes in both directions along the channel. The flow forms a stagnation point at the center and the effluence is restricted by a magnetic field. A theoretical analysis of steady state solutions of the MHD equations in the incompressible case is given as a function of three parameters: the Reynolds number Re, the magnetic Reynolds number Rm and Alfvenic Mach number MA for some of significant asymptotic limits. For highly conducting plasma (Rm >> 1) it was found that the magnetic field restrains the outflow for MA <1 and drives the escape for MA >1. In motions of low conductivity (Rm <<1) the magnetic field contains (and can be used for controlling) the effluence.**

Keywords **- Parallel porous walls; conducting fluids; Injection; Ejection.**

Resumen **- Una conducción de fluido se inyecta de forma continua o expulsada a través de un par de paredes porosas paralelas y se escapa en ambas direcciones a lo largo del canal. El flujo forma un punto de estancamiento en el centro y la emanación es restringida por un campo magnético. Un análisis teórico de soluciones de estado estacionario de las ecuaciones MHD en el caso incompresible se da como una función de tres parámetros: el número de Reynolds Re, el número de Reynolds magnético Rm y Alfvenic número de Mach MA para algunos de límites asintóticos significativos. Para conducir plasma (Rm >> 1) se encontró que el campo magnético restringe el flujo de salida para MA <1 y acciona el escape para MA> 1. En movimientos de baja conductividad (Rm << 1) el campo magnético contiene (y se puede utilizar para el control de) la emanación.**

Palabras clave **- Paredes paralelas porosas; conducción de fluidos; inyección; expulsión.**

I. Introduction

The movement of ordinary fluids that are injected or ejected by porous channels has been of considerable interest in recent literature on hydrodynamics. The bidimensional problem of a viscous and incompressible fluid in a porous channel with a stagnation point in the center was initially studied by Berman [1] whose work was motivated to give a model that explained the separation of uranium from U_{238} to U_{235} by gaseous diffusion. The uranium is previously turned to the gas UF_6, which has appropriate characteristics for its manipulation. In this pioneering work the problem of the stationary case was solved, using similar solutions to reduce from the Navier-Stokes equation to a differential equation of fourth degree, with a pair of border conditions in each wall. Berman found analytical solutions for the asymptotic situation of low Reynolds numbers in the case of suction in the walls. Later authors have studied different physical situations from this problem, Sellars [2], Yuang [3], Proudman [4], Shrestha [5], Terril [6], Brady and Acrivos [7], Brady [8], Robinson [9], Zaturska *et. al*.

[1] Producto derivado del proyecto de Investigación "Implementación de técnicas de modelamiento, procesamiento digital y simulación para el estudio de sistemas físicos", apoyado por las Universidades Tecnológica de Pereira y la Nacional sede Manizales, través del grupo de investigación: Laboratorio de Física del Plasma-Gednol.

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[10], Watson et al [11], Cox [12], Banks [13, 14] that in general has treated, for example the cases of symmetrical, asymmetric flows, walls with acceleration, different speeds from suction or injection in the walls superior and inferior. Taylor et al. [15], solved the three-dimensional problem of flows in porous channels, where the bidimensional case with cartesian and cylindrical geometries, are obtained like particular cases, by the variation of a parameter that gives the dimensional character of the problem. Since the case of injection is always temporarily stable, Hocking [16] paid special attention to the study of the stability in the case of suction. Solutions obtained showed that they are unstable for a critical Reynolds number R_e = 6.0014. For certain great values of Re, the flows have a periodic behavior from chaotic R_e= 12.936 and for $R_e > 20$.

In this work a conducting fluid which is continuously injected or ejected through a pair of parallel porous walls and which escapes in both directions along the channel is study. A theoretical analysis of the MHD equations steady state solutions in the incompressible case is given as a function of three parameters: the Reynolds number R_e, the *magnetic* Reynolds number R_m and Alfvenic Mach number M_A for some of significant asymptotic limits.

Basic Equations of the Magnetohydrodynamics Problem

The Navier-Stokes and the Ohm law, equations could be written in a reduced form as:

$$(\partial_t - \nu\nabla^2)\nabla^2\xi - \left[\xi,\nabla^2\xi\right] - \frac{1}{4\pi\rho}\left[\nabla^2\psi,\psi\right] = 0 \qquad (1)$$

$$(\partial_t - \nu_m\nabla^2)\psi - \left[\xi,\psi\right] = cE_z \qquad (2)$$

In the last equations, the velocity and the magnetic field are given by the following equations:

$$V_x = \partial_y\xi \quad V_y = -\partial_x\xi \quad B_x = \partial_y\psi \quad B_y = -\partial_x\psi \qquad (3)$$

The bracket $\left[f,g\right] = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}$ defines the Jacobian f of and functions, additionally $\nu_m = \frac{c}{4\pi\sigma}$ is the magnetic diffusion, $-\nabla^2\xi$ is the z component of the vorticity, $\boldsymbol{W} = \nabla\times\boldsymbol{V}$ ψ is the z component of the potential vector $\mathbf{A}$ $\nabla\times\mathbf{A} = \boldsymbol{B}$ which means, the J_z component can be written in terms of the function ψ like $-\nabla^2\psi = \frac{1}{4\pi}J_z$. The brackets $\left[\xi,\nabla^2\xi\right]$ and $\left[\xi,\psi\right]$ represent the convection transport terms of $-\omega_z$ and ψ_z, respectively. Additionally, the bracket $\left[\nabla^2\psi,\psi\right]$ represents the curl z component of the Lorentz force. We Suppose that there is an invariance of the translation in z, this is, for example,

$$\frac{\partial E_x}{\partial x} = 0 \text{ and } \frac{\partial E_y}{\partial y} = 0 .$$

Besides, $E_z = E_z(t)$ st depends on the time. In general the electrical field can be described by $\boldsymbol{E} = -\nabla\varphi + \boldsymbol{E}_z(t) - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}$ therefore it could be found a potential $\hat{\varphi}$, such that $\hat{\varphi} = \varphi(x,y,t) + E(t)z$.

We will study a model where the lateral walls, which are supposed to be distant in the z direction, cannot be charged electrically, then the z- component of the electrical field E_z is zero. This fact will permit us to see ahead, the use of similar solutions for uncoupling the equations (1) and (2). It can be supposed as well that those walls are conductive but they are in short-circuiting for a lab model (see fig. 1). Finally, these walls could be in the infinite, this last case is presented for instance in an astrophysic model. From above it can be deduced that in a tree-dimensional model the boundary conditions in z are related to the E_z electrical field.

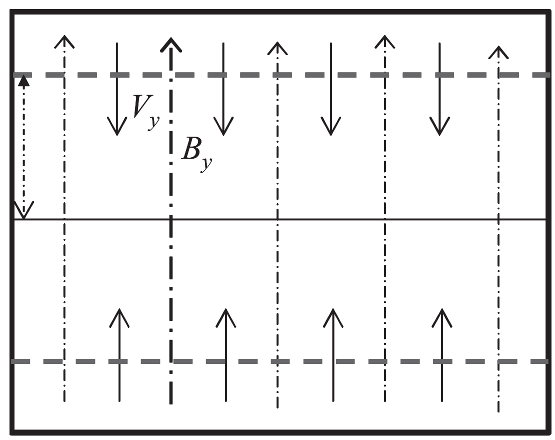

Fig 1. A conductor fluid injected through the walls. The magnetic field is represented without interaction between the field and the fluid.

The equations system (1) and (2) expounded previously, admits in general similar solutions of the form:

$$\xi = xf(y,t,\eta) + g(y,t,\eta) \qquad (4)$$

$$\psi = xp(y,t,\eta) + q(y,t,\eta) \qquad (5)$$

Where η represents all parameters involved, in this case, the viscosity, the magnetic diffusivity, and external magnetic field. Replacing the equations (4) and (5) in the equations (1) and (2),the following dimensionless equations system is obtained in the suction and injection cases of a fluid between two parallel and porous plates with an external magnetic field:

$$f_{tyy} - \frac{1}{R_e}f_{yyyy} = \left(ff_{yy} - (f_y)^2\right)_y - \frac{1}{M_A^2}\left(pp_{yy} - (p_y)^2\right)_y \qquad (6)$$

$$p_t - \frac{1}{R_m}p_{yy} = fp_y - f_y p \qquad (7)$$

R_e is the Reynolds number, R_m is the magnetic Reynolds number, H_a is the Hartmann number and M_A is the alfvenic Mach number. Notice that now $f = f(y,t,R_e,R_m,M_A)$ and $p = p(y,t,R_e,R_m,M_A)$. Given, R_e R_m and M_a, the system given by the equations (6) and (7) could be solved numeric or analytically in the asymptotic situation that we will study later. Sometimes, when the shooting technique is used in order to solve cases for the equations system given previously, the problem is invested and the parameters employed to make the calculations are R_e, R_m y M_A. Notice that if $p=0$ the problem decreases to the pure fluidness case. The general problem so expounded is quiet complex from a mathematical point of view. We will study the case in which a conductor, incompressible and viscous flow, goes in or out through a pair of parallel infinite perforated walls with the same suction rate or with injection in both walls (separated by a distance of $2h_0$). The flow that interacts with a magnetic field is basically perpendicular to the walls in the case of being conductors. If the walls are dielectric the magnetic field can have x and y components in the boundary. This magnetic field is modified by the conductor flow movement, as it is shown in figure 2.

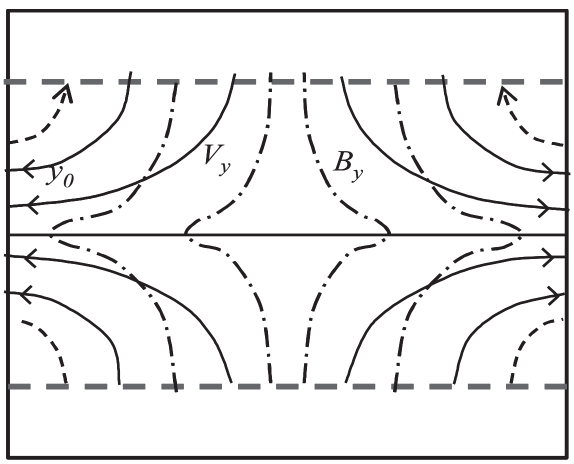

Fig. 2. Lines speed and the field in the case where it is considered that interaction with the magnetic field exists. This case comes when injection of flow through the walls exists.

The magnetic field $\mathbf{B} = (B_x, B_y, 0)$ and the speed $\mathbf{V} = (V_x, V_y, 0)$ could be obtained then of the following form:

$$V_x = \partial_y \xi = x f_x \qquad V_y = -\partial_x \xi = -f \tag{8}$$

$$B_x = \partial_y \psi = x p_y \qquad B_y = -\partial_x \psi = -p \tag{9}$$

II. Boundary Conditions

For the case of a fluid that enters or leaves for a pair of perforate and parallels walls, in presence of a magnetic field the initial conditions, are: $f(y,0,\eta) = f(y,\eta) = f_0(y,\eta) + f_1(t)$, here the temporary part $f_1(t)$ and,

$$p(y,0,\eta) = p(y,\eta) = p_0(y,\eta) + p_1(t)$$

here $p_1(t)$ are small interferences of f_0 y p_0, that they in turn are the solutions of the stationary case obtained from the equations (1) and (2). Additionally η is determined by a fixed parameters set of the system v_0, v_{0m} and B_0 which correspond to the values in the cinematic and magnetic viscosities and in the magnetic fields respectively. Here it is convenient to define the following operators:

$$Hf = \left[f(-1), f_y(-1), f(1), f_y(1) \right] \tag{10}$$

$$H_S f = \left[f(1), f_y(1), f(0), f_{yy}(0) \right] \tag{11}$$

$$H_A f = \left[f(1), f_y(1), f_y(0), f_{yyy}(0) \right] \tag{12}$$

$$Kp = \left[p(-1), p(1) \right] \tag{13}$$

$$K_\beta p = \left[p(1), p_y(1) \right] \tag{14}$$

The boundary condition for the velocity, the condition over f, in the case of suction in the walls is $Hf = [1,0,-1,0]$ and for the injection case $Hf = [-1,0,1,0]$. Since that in our numerical calculus we have integrated the equations (6) and (7) between the half of the channel width (y=0) and in the wall (y=1), we define the operators H_S and H_A which correspond to the cases of symmetric solutions ($H_S = [\pm 1,0,0,0]$ for injection (+) and suction (-)) and antisymmetric $H_A = [\pm 1,0,0,0]$ (for injection (+) and suction (-)) respectively. Note that if represents a symmetric flow then this should be an odd function so that $f(0) = 0$, and therefore the origin is always a stagnation point.

The boundary conditions for the magnetic field, the conditions over p_1 depend on the walls and flow conductor character. Nevertheless the following condition for both suction and injection cases, should be generally satisfied:

$$K_A p = \left[-1, \beta \right] \tag{15}$$

If the walls are conductors the constant β is adjusted depending on the characteristic parameters of the problem. For example, in the low viscosity and high conductivity regimes $\beta = 0$ is taken. Since in this case the flow drags the magnetic field lines so that they become parallel and therefore the magnetic field and the velocity satisfy the same boundary condition over the wall. This last condition, is valid for all the conductor walls. If the walls are dielectric then the magnetic field between them, is supposed to be originated by a pair of external coils that generate an external magnetic field in the form:

$$B_x^e = cx \qquad B_y^e = -cy + b \tag{16}$$

It means that the function that represents the magnetic field flow (equation (2)) is now given by the expression:

$$\psi^{ext} = -bx + cxy \tag{17}$$

Providing that the magnetic field in the walls can take any value, the boundary conditions for the magnetic field inside the walls that permit the couple with the external magnetic field, can be briefly written in the following way:

$$K_\alpha p = \left[p_y(1), p_y(0) \right] \qquad (18)$$

If $K_a = [c,0]$, the symmetric case of dielectric walls is obtained, but if $K_\alpha = [0,0]$, then the walls will be metallic. Since in this work we just present the symmetric flow case, the condition $f(\pm 1) = \mp 1$ is fixed for the flow and for the magnetic field $p(\pm 1) = -1$ and $R_m > 0$ for the numerical case, what means to take $R_e > 0$ and $R_m < 0$ for the suction case while for the injection case y , without varying the boundary conditions. Additionally when using the above convention, the time changes of sign in the injection case. It is clear that the negative time and negative Reynolds number definitions do not have any physical interpretation, it is just a mathematic artifice used in this kind of problems in order to facilitate the numerical calculus. In some cases it is convenient to use an integration of the equation (6). For the stationary case the equations system given in the equations (6) and (7) is described by the following equations system:

$$-\frac{1}{R_e} f_{yyy} = C + \left(ff_{yy} - (f_y)^2 \right) - \frac{1}{M_A^2} \left(pp_{yy} - (p_y)^2 \right) \qquad (19)$$

$$-\frac{1}{R_m} p_{yy} = fp_y - f_y p \qquad (20)$$

The constant C of integration is determined starting from the values in the boundary, that in stationary case of conductor walls and of injection of flowing in the walls, C is given by the equation:

$$C = \Big[\quad R_e \quad f_{yyy} \quad (f_{yy}) \qquad (21)$$

This integration constant C, on the other hand is directly related with the pressures gradient according to the x axis through the expression:

$$(\partial p / \partial x = -ax - xP'^2 / 4\pi) \qquad (22)$$

What is to say the pressures gradient depends not only on the x magnetic field component but on the position according to the x axes.

Asymptotic Approximations $R_E << 1$ and $R_M << 1$.

In the injection case with low Reynolds numbers, the magnetic field lines are now rigid just by a small perturbation which is caused by the flow movement, this one at the same time is very viscous for this limit ($R_e << 1$). Such magnetic field can be written in the following way:

$$p = p_0 + p_1, \qquad (23)$$

where p_0 is the field value that we assumed as constant and by simplification reasons can be taken the same as the unit. On the other hand, p_1 is a small perturbation which as it was previously said, it is caused by the fluid movement. Thus the equations (19) and (20) previously linearized, can be written in the following way: (see fig. 2),

$$-\frac{1}{R_e} f''' = C - \frac{1}{M_A^{\ 2}} p_0 p_1'' \qquad (24)$$

$$\frac{1}{R_m} p_1'' = f' p_0 \qquad (25)$$

In the expressions above the second order terms have been suppressed, that means, we have taken the first two terms of the expansion $p = 1 + R_m p_1 + \ldots$.

On the other side, the term $1/R_m$ is very big, but p_1'' is very small, so the equation (25) is valid. When replacing the equation (24) the following differential equation is obtained:

$$f''' = -CR_e + H_a^{\ 2} p_0^{\ 2} f' \qquad (26)$$

This equation at the same time has as solution (with $p_0 = 1$):

$$f = \left(y - \frac{Senh(H_a y)}{H_a Cosh(H_a)} \right) \bigg/ \left(1 - \frac{Tanh(H_a)}{H_a} \right) \qquad (27)$$

and consequently replacing the equation (25) the following expression for p_1 is obtained:

$$p_1 = -\frac{R_m((H_a y^2/2) - Cosh(H_a y)/H_a Cosh(H_a))}{H_a - Tanh(H_a)} + D \qquad (28)$$

Here, H_a is the Hartmann number defined previously. Also the integration constant D is calculated keeping in mind that the wall interference should be null, and then it remains defined like:

$$D = \frac{R_m(H_a/2 - 1/H_a)}{H_a - Tanh(H_a)} \qquad (29)$$

Figure 3 shows the velocity component behavior according to the x axis direction for different values of the Hartmann number. Note that when the Hartmann number grows , that means, the magnetic field becomes stronger ($M_A << 1$), the fluid behavior is similar to the Hartmann flow where the velocity is constant at the center of the channel and it strongly varies when is near the walls until diminishing to zero exactly over the wall.

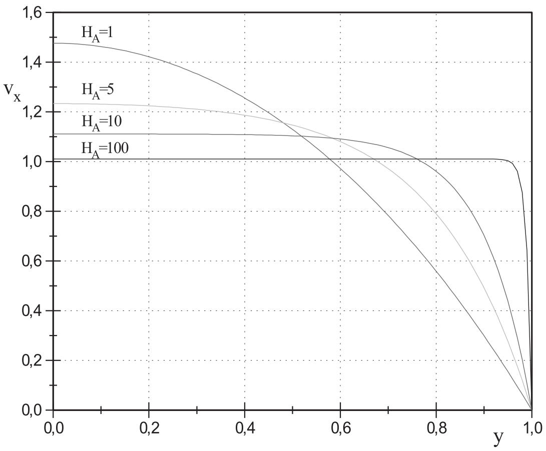

Fig. 3. Speed profile and their behavior for several values of the Hartmann number. It is observed that when Ha>>1 appears a limit layer in the wall.

On the other hand, taking into account the boundary condition in the wall *f(y*=1*)* =1, it is found that the constant *C* is related to the other constants through the following formula:

$$C = \frac{H_a^{\,3}}{R_e}\left(\frac{1}{H_a - \tanh(H_a)}\right) \tag{30}$$

Figure 4 shows the relation between the constants *C, H_a* and R_e given in the equation (30). Additionally if $H_a >> 1$ (for example, $M_A << (R_e R_m)^{(1/2)} << 1$), it implies that $C \approx H_a^2 / R_e$, so that it can be deduced that it should exist a strong gradient that moves the fluid outside. The magnetic field roughness controls then the fluid movement, avoiding it to leave. On the other hand if $H_a << 1$, the magnetic field lines are "less rigid " and in this case the condition $CR_e \approx 3$ is satisfied, thus the viscous effects are now the ones that control the fluid movement.

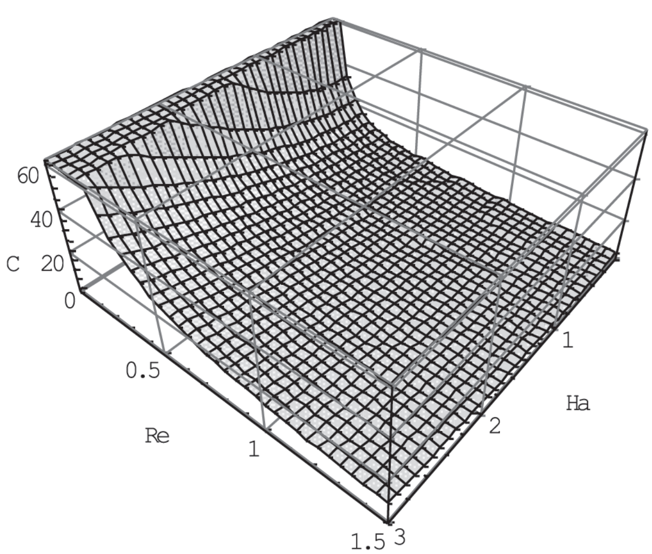

Fig. 4. Relation between C, Ha and Re in the asymptotic case of Rm<<1 and Re<<1.

On the other hand, figure 5 shows the current lines and the magnetic field obtained by the basic equations numerical integration. Note that the rigidity of the magnetic field lines, as well as the component B_x in the wall are not annulled. In this graphic it is difficult to see, due to the scale that it was built with.

Figure 6 illustrates the speed and field profiles, where the appearance of the limit layer before mentioned is shown. Similarly, how it was made in the previous asymptotic case, the solutions obtained upon being integrated numerically the equations (19) and (20) for the Runge-Kutta method, for the values $R_e = 0.1$, $R_m = 0.1$, $M_A = 0.3$ and $C = 31.3254$, they coincide with the obtained through the equation (30), where the value that is obtained is $C = 31.3326$. So it is shown again a good agreement between the asymptotic results and found numerals upon integrating the complete equations system. On the other hand, in the numeric integration that was made for several Reynolds number values, they do not show appreciable variations, for both profiles of the speed and the magnetic field, in the range $0.1 \leq R_e \leq 30$.

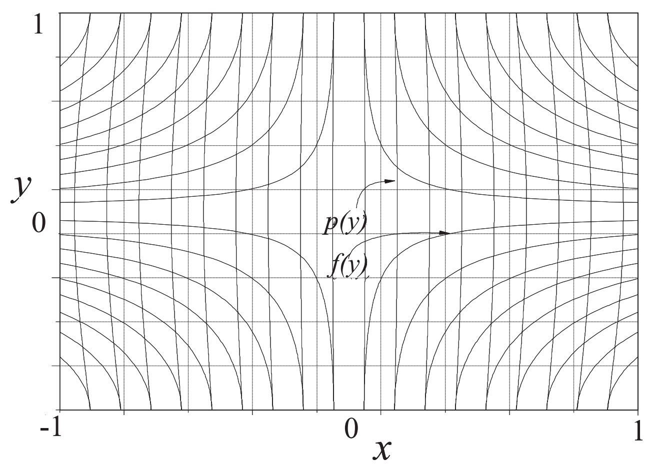

Fig. 5. For the asymptotic case, Rm<<1 y Re<<1, the field lines for the speed and the magnetic field is shown. In this case Rm=Re=0.1, MA=0.3 and C=31.3254.

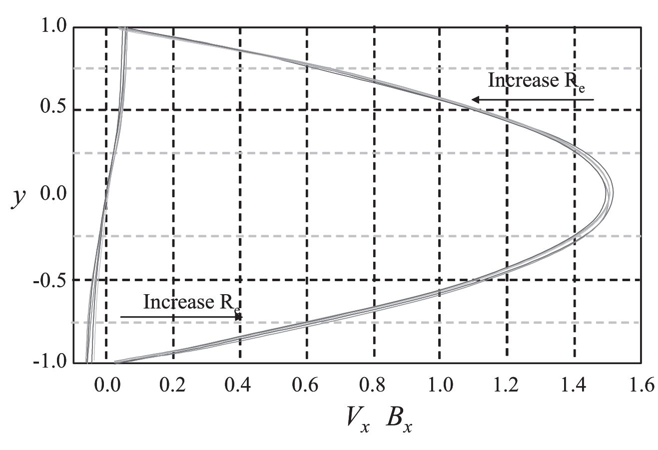

Fig. 6. For the case Rm<<1 and Re<<1, the profile of the speed and the magnetic field is shown. It is observed it that the field does is not null in the wall.

Solution with limit layer in the wall for R_E>>1, R_M>>1 and M_A<<1.

For the case given in the equations. (19) and (20) when R_e>>1, R_m>>1 and if $M_A{}^2 << 1$, the equations to solve now are:

$$C + \frac{1}{M_A{}^2}\left(p'^2 - pp''\right) \approx 0 \qquad (31)$$

$$-\frac{1}{R_m}p'' = fp' - pf' \qquad (32)$$

The equation (32) could be also written in the following form:

$$f = -\frac{p}{R_m}\int_0^y \frac{p''}{p^2}dy \qquad (33)$$

The equation (31) indicates essentially that the magnetic field is equilibrated with the pressure, and that the viscous and inertial effects are despicable, which leads to that the Navier–Stokes equation could be written in the following form:

$$\nabla P + \frac{1}{c}\mathbf{J}\times\mathbf{B} \approx 0 \quad \mathbf{V} \approx 0 \qquad (34)$$

The solution to the equation (31) is:

$$p = -\frac{\cosh(ky)}{\cosh(k)} \qquad (35)$$

Thus, with the boundary conditions $p(\pm 1) = -1$ and $B_x(\pm 1) = 0$, and with p given by the equation (35), it is obtained:

$$B_x = xp' = -kx\frac{senh(ky)}{\cosh(k)} \qquad (36)$$

Nevertheless, $B_x(\pm 1) \neq 0$ which shows that the walls are dielectric materials (or they are coated by a thin dielectric layer). Therefore for this regime there is no a solution for the conductor walls case, unless a limit layer in the wall is developed. The flow velocity consistent with the solution given in the equation (36) is described by the following relation:

$$-V_y = f(y) = \frac{k^2}{R_m}\cosh(ky)gd(y) \qquad (37)$$

Which in fact solves the equation (33) and additionally satisfies the stagnation point condition $f(0) = 0$. In the expression above the Gudermannian function $gd(y)$, represents the integral:

$$gd(y) = \int_0^y \frac{d\xi}{\cosh(k\xi)} \qquad (38)$$

As in the walls, in the injection case, $f(\pm 1) = 1$, k should be satisfied, this should be determined by the transcendental equation roots:

$$1 = \frac{k^2}{R_m}\mathrm{Cosh}(k)\int_0^1 \frac{d\xi}{\mathrm{Cosh}(k\xi)} \qquad (39)$$

$$f'(y) = -\frac{k^2}{R_m} + \tanh(ky)f(y) \qquad (40)$$

Then from the condition $f'(\pm 1) \neq 0$, it is observed that a viscous limit layer in the walls is always generated because the condition below is not satisfied,

$$V_x = xf' = 0 \qquad (41)$$

Suction case when R_E<<1 and R_M<<1.

In this case, the solutions are similar to those obtained for the injection case, just that now the boundary conditions change, the solution then for f is given by the expression:

$$f = -\left(y - \frac{Senh(H_a y)}{H_a Cosh(H_a)}\right) \Big/ \left(1 - \frac{Tanh(H_a)}{H_a}\right) \qquad (42)$$

Where it has been assumed that the magnetic field can be also written like $p=p_0+p_1$, so the perturbation p_1 for the magnetic field can be written in the following way (here we have also assumed that p_0=1):

$$p_1 = \frac{R_m((H_a y^2/2) - Cosh(H_a y)/H_a Cosh(H_a))}{H_a - Tanh(H_a)} + D \qquad (43)$$

As the perturbation has to be annulled in the wall, the constant D is then given by the expression:

$$D = -\frac{R_m(H_a/2 - 1/H_a)}{H_a - Tanh(H_a)} \qquad (44)$$

From above and being consistent with the equation (30), the integration constant C also changes sign:

$$C = \frac{H_a{}^3}{R_e}\left(\frac{1}{-H_a + Tanh(H_a)}\right) \qquad (45)$$

On the other side, figure 7 shows the current and magnetic field lines obtained from the numerical integration of the basic equations R_m=0.1, M_A=10 and R_e=0.1. Note that the magnetic field lines continue rigid but this is due to the fluid suction by the walls, these lines curvatures are opposite to the ones of the injection case.

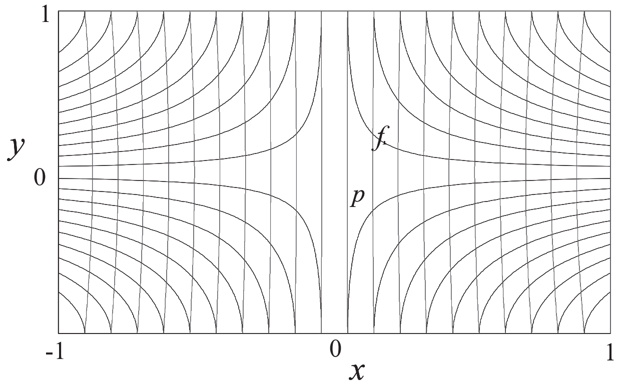

Fig. 7. Contour lines of the speed and the magnetic field for Rm=0.1, MA=10 and Re=0.1. The magnetic lines flexion is contrary to those obtained for the injection case.

Additionally for the same values above of M_A=10, and R_m=0.1, the velocity and magnetic field profiles in this asymptotic regime, are shown in fig. 8. In this figure the R_e values are shown, varying between 0.3 and 27.3 and with an increase of 3. As it can be seen, as the flow becomes less viscous, the magnetic field effect becomes more notorious, doing that the fluid velocity existing in the center of the channel diminishes appreciably and as consequence, flows with much more velocity appears near the walls. As this one is nearer the walls, the velocity change is going to be bigger, for example in y=0.96, the v_x velocity varies from 0.4 to 0.8, when the Reynolds number changes from 0.1 to 30. Nevertheless, the x magnetic field component increases its value in the wall showing in this way a limit layer apparition, in the conductor walls case, or the walls could be dielectric (at least if they are coated by a thin dielectric layer). The above is reasonable if it is taken into account that the fluid has to come from a deposit placed far from the center of the channel and it has to come out by the walls, but due to the strong magnetic field and as the fluid becomes less viscous, this one flows less by the center of the channel. In this point it is convenient to clarify that in our numerical calculus we have omitted the condition p' equals zero in the wall and this value is let to adjust it freely to the other problem conditions.

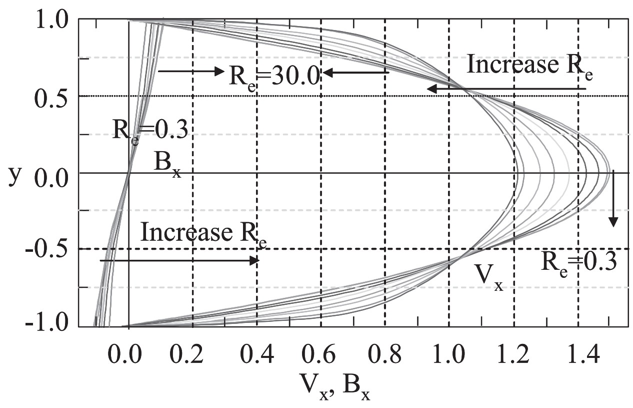

Fig. 8. Speed profile behavior and magnetic field as a function of the Reynolds number Re for Rm=0.1, and MA=10.

On the other side figure 9 shows the vorticity values in the wall in function of the R_e Reynolds number, when R_m=0.1 and M_A=10 is taken. It is seen that in this figure, the vorticity varies very slowly for the injection case and it considerably increases in the suction case as the Reynolds number grows, showing in this way a limit layer apparition for the case R_e>>1. Additionally if the magnetic field effect is compared regarding to the pure fluid case, where it is seen that in injection, the vorticity in the wall slightly increases while than for suction case, such vorticity considerably diminishes as the Reynolds number grows. It is important to say that when the Reynolds number is null then f'' =2.984 with or without magnetic field.

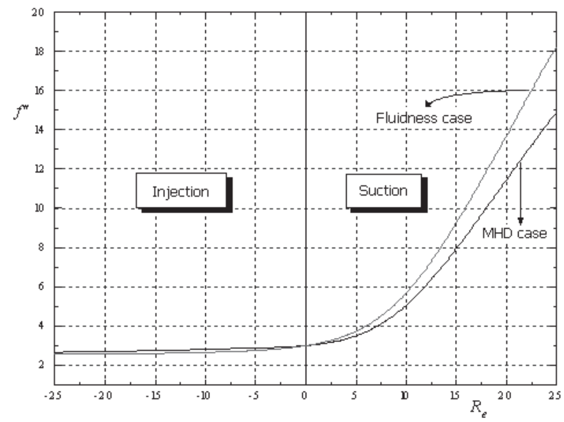

Fig. 9. Graphic of the vorticity in the wall as a function of the Reynolds number. It is observed that for the suction case the effect of the magnetic field makes that the vorticity in the wall disappears.

III. Conclusions

A theoretical analysis of the steady state solutions of MHD equations in the incompressible case is given as a function of the Reynolds number R_e, the magnetic Reynolds number R_m and Alfvenic Mach number M_A for some of significant asymptotic limits has been used for a conducting fluid which is continuously injected or ejected through a pair of parallel porous walls and escapes in both directions along the channel. When the fluid is symmetric, the velocity is represented by a symmetric function and the center of the channel is a stagnation point. When Re<<1 and Ma<<1, the magnetic field lines are a little curved towards the center of the cannel, in the suction case and moving away of the center in the injection case. The magnetic field controls roughly the fluid movement, avoiding it to leave. On the other hand if $H_a << 1$, the magnetic field lines are "less rigid " and the viscous effects are now the ones that control the fluid movement and appears a limit layer in the wall.

When Ha>>1, the fluid velocity remains almost constant in the center of the channel and it has strong variations close to the walls until diminishing to zero exactly over the wall.

For fixes values of M_A and R_m when Re increases as the flow becomes less viscous, the magnetic field effect becomes more notorious, doing that the exit velocity of the fluid in the channel center diminishes appreciably and as consequence, it flows with more velocity near the walls. The *x* magnetic field component increases its value in close to the wall showing in this way a limit layer apparition, in the conductor walls case.

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