

LMI sliding mode control approach for electromechanical systems¹

Técnica de control LMI-modo deslizantes para sistemas electromecánicos

J.M. Salamanca, R.A. Plazas and E.J. Avella

Recibido Marzo 01 de 2014 – Aceptado Noviembre 20 de 2014

Abstract - This paper presents a control approach that is based on the combination of Linear Matrix Inequalities and Sliding mode control. This approach is appropriate to electromechanical systems like robotic manipulators, helicopter models, inverted pendulum systems and so. The proposed control law is has two parts: a linear component and a nonlinear component. The linear component is designed with the objective to track an appropriate reference model given. The nonlinear control component provides robustness in face nonlinearities, disturbances, parametric uncertainties and sensor faults. The performance of proposed controller is tested in two examples: the PR and ELBOW robot models, which are simulated in Simulink of MATLAB®.

Key Words - linear matrix inequalities, sliding modes, robot manipulators, sensor fault.

Resumen - Este artículo presenta una técnica basada en la combinación de desigualdades matriciales lineales y control con modos deslizantes. Esta técnica es apropiada para sistemas electromecánicos como manipuladores robóticos, modelos de helicópteros y sistemas de péndulo invertido. La ley de control propuesta tiene dos partes: una componente lineal y una componente no lineal. La componente lineal es diseñada con el objetivo de seguir un modelo de referencia establecido. La componente de control no lineal proporciona robustez frente a no-linealidades, perturbaciones, incertidumbres paramétricas

y falla de sensores. El desempeño del controlador propuesto es probado en dos ejemplos, los modelos de robot: PR y ELBOW, utilizando Simulink de MATLAB®.

Palabras Clave: desigualdades matriciales lineales, modos deslizantes, robot manipuladores, falla de sensores.

I. INTRODUCTION

THERE are nonlinear systems like robotic manipulators [1], helicopter models [2], [3], among other, which have some characteristics that allow to apply hybrid techniques as the combination of Linear Matrix Inequalities and Sliding Mode Control (LMI-Sliding Mode Control). Several approaches of control have been applied to robotics. Approaches as PID, Nonlinear with Observer, Robust H_∞ , Fuzzy control as it can see [4], [5], [6], [7], [8], [9] shows some efficiency in the performance of the manipulators. But those control laws are complex and difficult to implement. The control approach proposed in this paper allows us to control nonlinear system with matched and unmatched uncertainties and disturbances [10]. Nonlinear systems with unmodeled dynamics can be controlled by using the LMI-Sliding Mode Control approach, as show in [11], [12], [13], [14], [15]. The complexity of mechatronic systems causes increasing susceptibility to faults, especially sensor faults. LMI-Sliding Mode Control approach allows take into account sensor faults/ in control applications.

The paper is organized as follows. The section II describes a model generic and its expression to apply the control law. The section III describes the control objectives and the components of the control approach. In section IV, the linear control component design is showed. In section V, the nonlinear control part is designed. In VI section, the proof of the global stability is realized. In section VII, the application of the control law to the robots PR and ELBOW

¹ This work was supported for processing signal research group DSP-UPTC with DIN-Uptc (Dirección de Investigaciones de la Universidad Pedagógica y Tecnológica de Colombia) Project No 806.

J. Salamanca is with the Department of Electronic Engineer, Universidad Pedagógica y Tecnológica de Colombia. Calle 4 sur # 15-134 Sogamoso, Colombia (Email: juan.salamanca@uptc.edu.co)

R.A Plazas-Rosas (Email: ramiro.plazas@uptc.edu.co)

E.J Avella-Rodríguez (Email: ramiro.plazas@uptc.edu.co)

examples is showed. Finally the section VIII concludes the paper.

II. MODELING

Many electromechanic systems like robots manipulators, rotational pendulums and helicopter models can be modeled by the following equations:

$$\begin{aligned}\dot{X}(t) &= F(x) + G(x)U(t) \\ Y(t) &= CX(t)\end{aligned}$$

Where: $X(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$;

$$U(t) = [u_1(t) \ u_2(t)]^T$$

$$F(x) = \begin{bmatrix} x_3(t) \\ x_4(t) \\ F_3(x_1, x_2, x_3, x_4) \\ F_4(x_1, x_2, x_3, x_4) \end{bmatrix}$$

$$G(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ g_{31}(x_1, x_2, x_3, x_4) & g_{32}(x_1, x_2, x_3, x_4) \\ g_{41}(x_1, x_2, x_3, x_4) & g_{42}(x_1, x_2, x_3, x_4) \end{bmatrix}$$

$F(x)$ Is Lipchitz locally.

The nonlinear model can be expressed as:

$$\begin{aligned}X(t) &= AX(t) + BU(t) + \xi(x, u) \\ Y(t) &= CX(t)\end{aligned}$$

Where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi(x, u) = \begin{bmatrix} 0 \\ \xi_2(x, u) \end{bmatrix};$$

$$\xi(x, u) : \mathfrak{N}^6 \rightarrow \mathfrak{N}^4; \quad \xi_2(x, u) : \mathfrak{N}^6 \rightarrow \mathfrak{N}^2$$

$$A = \begin{bmatrix} 0 & I_2 \\ A_{21} & A_{22} \end{bmatrix}; \quad A_{21} = \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (8)$$

$$B_2 = \begin{bmatrix} b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \quad \text{Is an invertible matrix}$$

On $\xi_2(x, u)$ is a known condition

$$\|\xi_2(x, u)\|_2 \leq \alpha_2(x) + \beta_2 \|\phi U(t)\|_2 \quad (1)$$

$\beta \|\phi B_2^{-1}\| < 1$ In a region D_1 of R^4 that contains the origin

$\alpha_2(x)$ Is a nonnegative known function of state or a state part

β_2 Is a positive known constant

ϕ Is a matrix known that select a part of control

III. CONTROL OBJECTIVES

The control objective is the model described by (1) must be to track the reference model described by (10):

$$\begin{aligned}\dot{X}_m(t) &= A_m X_m(t) + B_m R(t) \\ Y_m(t) &= C_m X_m(t)\end{aligned} \quad (10)$$

Where $X_m(t) \in \mathfrak{R}^4$; $R(t) \in \mathfrak{R}^2$ and

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_{01}^2 & 0 & -2\rho_1\omega_{01} & 0 \\ 0 & -\omega_{02}^2 & 0 & -2\rho_2\omega_{02} \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \omega_{01}^2 & 0 \\ 0 & \omega_{02}^2 \end{bmatrix}$$

To achieve reach the model tracking it is necessary to meet the following conditions [16].

$$\begin{aligned}\text{rank}[A - A_m \quad B] &= \text{rank}(B) \\ \text{rank}[B \quad B_m] &= \text{rank}(B)\end{aligned} \quad (11)$$

Due to $\text{rank}[A - A_m \quad B] = 2$

$\text{rank}[B \quad B_m] = 2$, The conditions (11) are satisfied. (6)

The control law has two parts, one linear part and one nonlinear part (12). The linear one allows tracking the reference model and is obtained by using the linear model without uncertainties and disturbances. The nonlinear one aims to provide robustness in face to parametric uncertainties,

nonlinearities and disturbances. This component uses a nonlinear model and doesn't include the linear one. The robustness and stability properties are based on Lyapunov theory

$$U(t) = U_l(t) + U_n(t) \quad (12)$$

$U_l(t)$ The linear control component

$U_n(t)$ The nonlinear control component

IV. DESIGN OF LINEAR CONTROL COMPONENT

To design the linear control component a linear model is used without parametric disturbances and uncertainties. This model is described by:

$$\dot{X}(t) = AX(t) + BU_l(t) \quad (13)$$

$$\text{Where: } U_l(t) = -KX(t) + GR(t) \quad (14)$$

In closed loop the system rests:

$$\dot{X}(t) = (A - BK)X(t) + BGR(t) \quad (15)$$

By comparing the model (10) and the equation (15) it obtains the design equations:

$$A - BK = A_m \Rightarrow BK = A - A_m; \quad BG = B_m \quad (16)$$

$$K \in \mathfrak{R}^{2 \times 4}; \quad G \in \mathfrak{R}^{2 \times 2}$$

From equation (16), the solutions to K and G are unique. Now we define the tracking error: $\tilde{X}(t) = X(t) - X_m(t)$. And the error dynamic is:

$$\dot{\tilde{X}}(t) = A_m \tilde{X}(t) + \xi(x, u) \quad (17)$$

To analyze the stability of the closed loop system the Lyapunov stability theory is used as is presented in [17].

Be $V(\tilde{X}) = \frac{\tilde{X}^T P \tilde{X}}{2}$ a Lyapunov candidate function

Where $P = P^T > 0$; $P \in \mathfrak{R}^{4 \times 4}$

$$V(\tilde{X}) > 0; \quad \forall \tilde{X} \in \mathfrak{R}^4; \quad \tilde{X} \neq 0$$

The model A_m has its Eigenvalues located in the left half plane of S plane; hence, a linear matrix inequality (LMI) is satisfied according to [18], [19]:

$$A_m^T P + P A_m < 0; \quad P = P^T$$

With the overall control, the tracking error model rests:

$$\dot{\tilde{X}}(t) = A_m \tilde{X}(t) + BU_n(t) + \xi(x, u) \quad (18)$$

With this error dynamics, the energy function is:

$$V(\tilde{X}) = \tilde{X}^T \frac{(A_m^T P + P A_m)}{2} \tilde{X} + \tilde{X}^T P B U_n + \tilde{X}^T P \xi(x, u)$$

Be:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad P_{11} = P_{11}^T > 0; \quad P_{11} \in \mathfrak{R}^{2 \times 2} \quad (19)$$

$$P_{12} \in \mathfrak{R}^{2 \times 2}; \quad P_{21} \in \mathfrak{R}^{2 \times 2}; \quad P_{22} = P_{22}^T > 0;$$

$$P_{22} \in \mathfrak{R}^{2 \times 2}; \quad P_{12} = P_{21}^T$$

In many electromechanical systems the matrix B can be expressed as:

$$B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}; \quad \text{Where: } B_2 \in \mathfrak{R}^{2 \times 2} \text{ and is an invertible matrix}$$

V. NONLINEAR CONTROL COMPONENT DESIGN

In this section the error model is used to design the nonlinear component of the control.

$$\dot{\tilde{X}}(t) = BU_n(t) + \xi(x, u) \quad (20)$$

The objective of this component is to provide robustness with respect to parametric uncertainties and disturbances.

$$\tilde{X}_1(t) = [\tilde{x}_1(t) \quad \tilde{x}_2(t)]^T; \quad \tilde{X}_2(t) = [\tilde{x}_3(t) \quad \tilde{x}_4(t)]^T$$

The approach uses a surface of state space defined as:

Then the sliding mode surface is:

$$Z(t) = \tilde{X}_2(t) + P_{22}^{-1} P_{12}^T \tilde{X}_1(t) \quad (21)$$

From the nonlinear model described by (20), the dynamic of

$$Z(t) = \tilde{X}_2(t) + P_{22}^{-1} P_{12}^T \tilde{X}_1(t) = 0 \quad (22)$$

tracking error is:

The dynamic on the surface Z(t) is:

$$\dot{\tilde{X}}_1(t) = 0$$

$$\tilde{X}_2(t) = B_2 U_n(t) + \xi_2(x, u) \quad (23)$$

To obtain the nonlinear control component a Lyapunov

$$\dot{Z}(t) = B_2 U_n(t) + \xi_2(x, u) \quad (24)$$

candidate function is used. This Lyapunov function is:

$$V(Z) = \frac{Z^T(t) \Gamma Z(t)}{2}; \quad \text{Where } \Gamma = \Gamma^T > 0; \quad \Gamma \in \mathfrak{R}^{2 \times 2}$$

The sliding condition used from [20] is described by:

$$\dot{V}(Z) \leq -\eta \|\Gamma Z(t)\|; \quad \eta > 0$$

$$\text{Be: } \tilde{\rho}(x, U_l) = \eta + \alpha_2(x) + \beta_2 \|\phi U_l(t)\| \quad (25)$$

The nonlinear control component proposed is

$$U_n(t) = -B_2^{-1} (x, U_l) \frac{\Gamma Z(t)}{\|\Gamma Z(t)\|}; \quad Z(t) \neq 0 \quad (26)$$

The total control law is expressed in equation (27).

$$U(t) = -KX(t) + GR(t) - \frac{B_2^{-1} \tilde{\rho}(x, U_l)}{1 - \beta_2 \|\phi B_2^{-1}\|} \left[\frac{P_{22} Z(t)}{\|P_{22} Z(t)\|} \right] \quad (27)$$

With this control law, the condition $\dot{V}(\tilde{X}) < 0$ is obtained in some region $D \subset \mathbb{R}^4$ that contains the origin. This region depends of the region D_1 defined in (9). The control law allows to track the model described by (10) in despite nonlinearities disturbances and parametric uncertainties.

VI. GLOBAL STABILITY PROOF

Theorem 1: Given the model described by (1) with the specifications (2), (3) and the conditions given by (9) then the control law expressed by (27) establishes the asymptotic stability of the equilibrium point $\tilde{X}(t) = 0$ where $\tilde{X}(t)$ is the tracking error between the reference model state and the state of the model given by (1) in despite of parametric uncertainties and disturbances that affects the system.

Proof

Be the Lyapunov function candidate:

$$V(\tilde{X}) = \frac{\tilde{X}^T P \tilde{X}}{2}; \quad P = P^T > 0; \quad P \in \mathbb{R}^{4 \times 4} \quad (28)$$

Its derivative is then:

$$\dot{V}(\tilde{X}) = \tilde{X}^T \left(\frac{A_m^T P + P A_m}{2} \right) \tilde{X} + \tilde{X}^T P B_0 U_n + \tilde{X}^T P \xi(X, U) \quad (29)$$

Where:

$$\tilde{X}(t) = \begin{bmatrix} \tilde{X}_1(t) \\ \tilde{X}_2(t) \end{bmatrix}; \quad \tilde{X}_1(t) \in \mathbb{R}^2; \quad \tilde{X}_2(t) \in \mathbb{R}^2$$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} > 0$$

$$\tilde{X}^T P \xi(X, U) = (\tilde{X}^T P_{12} P_{22}^{-1} \tilde{X}_2) P_{22} \xi_2(X, U)$$

Replacing $U_n(t)$ by (26) and taking $\Gamma = P_{22}$ it obtains:

$$\dot{V}(\tilde{X}) = \tilde{X}^T \left(\frac{A_m^T P + P A_m}{2} \right) \tilde{X} - \frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z(t)\| + Z^T \Gamma \xi_2(X, U)$$

It is necessary to demonstrate the condition:

$$\frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z(t)\| - Z^T(t) \Gamma \xi_2(X, U) \geq 0$$

For all $X \in D$ at least. Because is known

$$\tilde{X}^T \left(\frac{A_m^T P + P A_m}{2} \right) \tilde{X} < 0; \quad \forall \tilde{X} \neq 0 \text{ Then:}$$

$$\begin{aligned} & \frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z\| - Z^T \Gamma \xi_2(X, U) \leq \\ & \frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z\| - [\alpha_2(X) + \beta_2 \|\phi_2 U\|] \|\Gamma Z\| \leq \\ & \frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z\| - [\alpha_2(X) + \beta_2 \|\phi_2 U_l\| + \beta_2 \|\phi_2 U_n\|] \|\Gamma Z\| \geq \\ & \tilde{\rho}(X, U_l) - (1 - \beta_2 \|\phi_2 B_{02}^{-1}\|) \alpha_2(X) - \\ & (1 - \beta_2 \|\phi_2 B_{02}^{-1}\|) \beta_2 \|\phi_2 U_l\| - \beta_2 (1 - \beta_2 \|\phi_2 B_{02}^{-1}\|) \\ & \left[\frac{\phi_2 B_{02}^{-1} \tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \right] \frac{\|\Gamma Z\|}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \\ & [\tilde{\rho}(X, U_l) - \alpha_2(X) - \beta_2 \|\phi_2 U_l\|] \|\Gamma Z\|; \\ & \eta \|\Gamma Z\| \geq 0; \text{ in } D \subset \mathbb{R}^4 \end{aligned}$$

Then

$$\frac{\tilde{\rho}(X, U_l)}{1 - \beta_2 \|\phi_2 B_{02}^{-1}\|} \|\Gamma Z\| Z^T \Gamma \xi(X, U) \geq 0; \text{ in } D \subset \mathbb{R}^4$$

Hence $\dot{V}(\tilde{X}) < 0$ for all $\tilde{X} \in D - \{0\}$ and the proof is concluded.

VII. APPLICATION EXAMPLES

A) ROBOT RP

The robot PR as is shown in Fig 1, is constituted by one rotational junction and one prismatic, hence this robot has two degrees of freedom (rotational and translational motions).

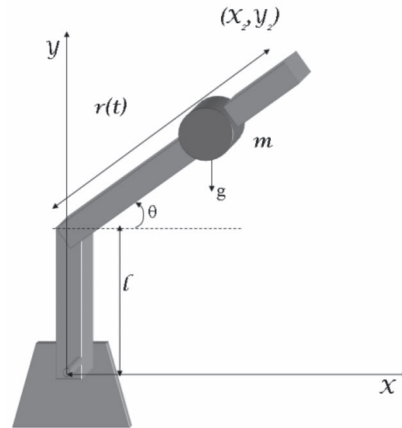


Fig. 1 Dynamics of RP robotic arm

The state variables are defined as:

$$x_1(t) = \theta_1(t); \quad x_2(t) = \theta_2(t); \quad x_3(t) = \dot{\theta}_1(t);$$

$$x_4(t) = \dot{\theta}_2(t)$$

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\begin{aligned}\dot{x}_3 &= \frac{-g \cos(x_1)}{x_2} + \frac{-2x_3x_4}{x_2} + \frac{1}{mx_2^2}u_1 \\ \dot{x}_4 &= -g \cos(x_1) + x_2x_3^2 + \frac{1}{m}u_2 \\ \phi &= [1 \quad 0]\end{aligned}\quad (31)$$

Taking the model (4)

$$\xi(X, U) = \begin{bmatrix} 0 \\ 0 \\ \frac{-g \cos(x_1)}{x_2} - 2\frac{x_3x_4}{x_2} + \frac{1}{m}\left(\frac{1}{x_2^2} - b_0^2\right) \\ g(x_1 - \text{sen}(x_1)) + x_2x_3^2 \end{bmatrix}\quad (32)$$

$$\|\xi(X, U)\| \leq \frac{g}{x_{2\min}} + \frac{2|x_3||x_4|}{x_{2\min}} + \frac{1}{m} \frac{2|1 - b_0^2x_2^2||u_1|}{x_2^2} + 2g|x_1| + |x_2|x_3^2\quad (33)$$

$$B_{02} = \begin{bmatrix} \frac{b_0^2}{m} & 0 \\ 0 & \frac{1}{m} \end{bmatrix}\quad (34)$$

$$\|\phi_2 B_{02}^{-1}\| = \frac{m}{b_{02}}\quad (35)$$

$$\beta_2 \|\phi_2 B_{02}^{-1}\| = \frac{1 - b_0^2 x_{2\min}^2}{b_0^2 x_{2\min}^2}\quad (36)$$

$$1 < 2b_0^2 x_{2\min}^2 \Rightarrow x_{2\min} > \frac{1}{(\sqrt{2}b_0)} \text{ And the condition on}$$

$x_2(t)$ be:

$$\frac{1}{(\sqrt{2}b_0)} < x_2(t) < \frac{1}{b_0}\quad (37)$$

From the equation (16) the linear controller parameters are obtained as:

$$\begin{aligned}k_{11} &= \frac{m\omega_{01}^2}{b_0^2}; & k_{12} &= 0; & k_{13} &= \frac{2\rho_1\omega_{01}m}{b_0^2}; & k_{14} &= 0 \\ k_{21} &= -gm; & k_{22} &= \omega_{02}^2; & k_{23} &= 0; & k_{24} &= 2\rho_2\omega_{02}m \\ g_{11} &= \frac{m\omega_{01}^2}{b_0^2}; & g_{12} &= 0; & g_{21} &= 0; & g_{22} &= m\omega_{02}^2\end{aligned}$$

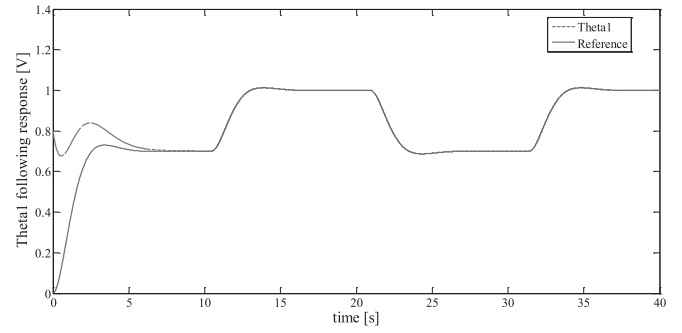


Fig. 2 Theta1 following response LMIs-based sliding mode.

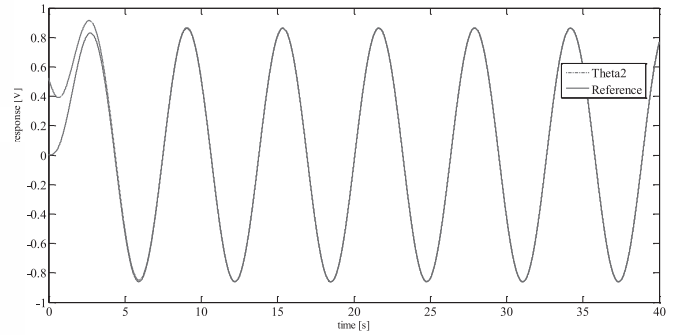


Fig. 3 Theta2 following response LMIs-based sliding mode.

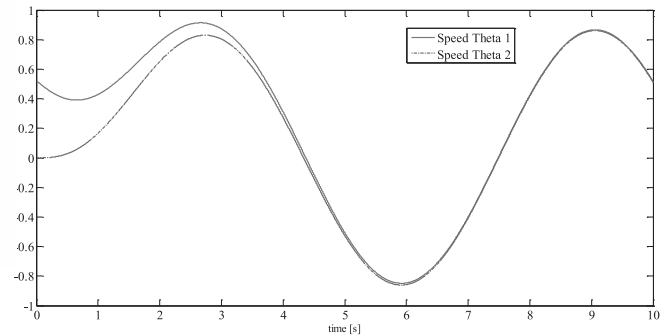


Fig. 4. Speed signals concerning the control-based sliding mode LMIs.

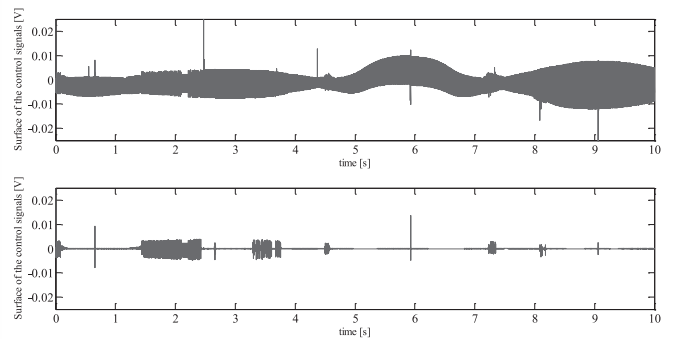


Fig. 5 Surface of the control signals based on LMIs-sliding modes.

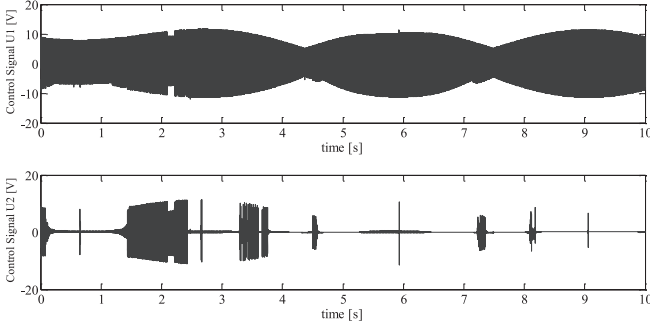


Fig. 6 Control signals concerning the control-based sliding mode LMIs.

In the Fig. 2 and 3, it observes the tracking performance of the position reference signals, in this example the reference signals used were a square wave for de variable X_1 and sinusoidal for the variable X_2 . The simulation shows tracking errors tend to zero in despite of parametric uncertainties and disturbances. It observes that the tracking of the variables X_{m1} and X_{m2} is satisfactory. The similar results are obtained for the variables of angular speed of the robot (see Fig. 4). In the Fig. 5 sliding surface relates to \tilde{X}_1 and \tilde{X}_2 are showed and it observes the little variation with respect to zero. In the Fig. 6 it observes the control signals. This signals show a high switching. This is characteristic of the sliding mode control. But the sizes of the control signals lie in practical acceptable levels.

B) ROBOT ELBOW

The Elbow robot [1] as is shown in Fig. 7 consists of two junctions, two arms with parameters from [21]:

$$a_1 = 0.20m; a_2 = 0.50m; m_1 = 1Kg; m_2 = 0.25Kg; \\ g = 9.8m/s^2$$

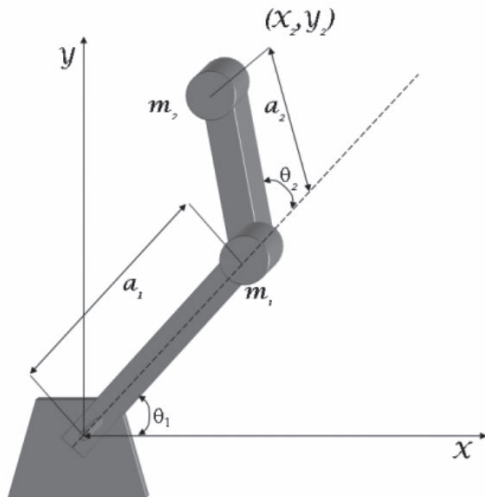


Fig. 7 Dynamics of 2-DOF robotic arm.

The state model is given by:

$$\begin{aligned} x_1(t) &= \theta_1(t); x_2(t) = \theta_2(t); x_3(t) = \dot{\theta}_1(t); \\ x_4(t) &= \dot{\theta}_2(t) \\ \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= \frac{a_3}{b_3} + \frac{u_1}{b_3} + \frac{c_3}{d_3} - \frac{e_3}{b_3} u_2 \\ \dot{x}_4 &= \frac{a_4}{c_4} + \frac{b_4}{c_4} u_1 + \frac{d_4}{f_4} - \frac{e_4}{f_4} u_2 \end{aligned} \quad (38)$$

Where:

$$\begin{aligned} a_3 &= (-m_2 a_1 a_2 \text{sen}(x_2) x_4^2 - 2m_2 a_1 a_2 \text{sen}(x_2) x_3 x_4 \\ &\quad - (m_2 a_2 g \cos(x_1 + x_2) + (m_1 + m_2)(a_1 g \cos(x_1) \\ &\quad + (a_2^2(1 + a_1 \cos(x_2)/a_2))m_2 g \cos(x_1 + x_2))) \end{aligned}$$

$$b_3 = m_1 a_1^2 (1 + (m_2/m_1) \text{sen}^2 x_2)$$

$$c_3 = a_2^2 (1 + (a_1/a_2) \cos(x_2)) m_2 \text{sen}(x_2) x_3^2$$

$$d_3 = m_1 (1 + (m_2/m_1) \text{sen}^2 x_2)$$

$$e_3 = a_2 (1 + (a_1/a_2) \cos(x_2))$$

$$\begin{aligned} a_4 &= (a_2 (1 + (a_1/a_2) \cos(x_2)) (-m_2 a_1 a_2 \text{sen}(x_2) x_4^2 - \\ &\quad 2m_2 a_1 a_2 \text{sen}(x_2) x_3 x_4) + (a_2 (1 + (a_1/a_2) \cos(x_2))) \\ &\quad m_2 a_2 g \cos(x_1 + x_2) + (m_1 + m_2)(a_1 g \cos(x_1))) \end{aligned}$$

$$b_4 = a_2 (1 + (a_1/a_2) \cos(x_2))$$

$$c_4 = m_1 a_1^2 (1 + (m_2/m_1) \text{sen}^2 x_2)$$

$$\begin{aligned} d_4 &= (-m_1/m_2 a_2 + 1/a_2^2 1/a_1^2 + (2 \cos(x_2)/(a_1 a_2))) \\ &\quad (m_2 a_1 a_2 \text{sen}(x_2) x_3^2) - (m_1/(m_2 a_2) + 1/a_2^2 1/a_1^2 \\ &\quad + (2 \cos(x_2)/(a_1 a_2))(m_2 a_2 g \cos(x_1 + x_2))) \end{aligned}$$

$$e_4 = (m_1/(m_2 a_2) + 1/a_2^2 1/a_1 + (2 \cos(x_2)/(a_1 a_2)))$$

$$f_4 = m_1 (1 + (m_2/m_1) \text{sen}^2 x_2)$$

The nonlinear model (39) was simulated in an S-function created in MATLAB®/Simulink. The nonlinear model is:

$$\dot{X} = A_0(x) + B_0 U + \xi(x, u) \quad (39)$$

Where:

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}$$

$$b_{31} = 1/(m_1 a_1^2) \quad b_{32} = -a_2/(m_1 a_1^2) \quad b_{41} = -a_2/(m_1 a_1^2);$$

$$b_{42} = (1/(m_1 a_1^2) + 1/(m_1 a_2^2) + 1/(m_2 a_2^2))$$

$$\|\xi_2(x, u)\|_2 \leq \alpha_2(x) + \beta_2 \|\phi U(t)\|_2; \quad \beta \|\phi B_2^{-1}\| < 1$$

Where:

$\alpha_2(x)$: Is a positive function of the state or the state.

β_2 : Is a positive constant.

ϕ : It is an identity matrix.

$$\xi(x, u) = \begin{bmatrix} 0 & 0 & \xi_3 & \xi_4 \end{bmatrix}'$$

With:

$$\xi_3 = a_3/b_3 + c_3/d_3 + (1/b_3 - 1/b_{30})u_1 + (e_3/b_3 - e_{30}/b_{30})u_2$$

$$\xi_4 = a_4/c_4 + d_4/f_4 + (b_4/c_4 - b_{40}/c_{40})u_1 + (e_4/f_4 - e_{40}/f_{40})u_2$$

$$\phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$k_{11} = \frac{b_{42} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}}; \quad k_{12} = \frac{b_{32} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}};$$

$$k_{13} = -2 \frac{b_{42} r_{o1} W_{o1}}{-b_{31} b_{42} + b_{32} b_{41}}; \quad k_{14} = 2 \frac{b_{32} r_{o2} W_{o2}}{-b_{31} b_{42} + b_{32} b_{41}}$$

$$k_{21} = \frac{b_{41} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}}; \quad k_{22} = \frac{b_{31} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}};$$

$$k_{23} = 2 \frac{b_{41} r_{o1} W_{o1}}{-b_{31} b_{42} + b_{32} b_{41}}; \quad k_{24} = -2 \frac{b_{31} r_{o2} W_{o2}}{-b_{31} b_{42} + b_{32} b_{41}}$$

$$g_{11} = \frac{b_{42} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}}; \quad g_{12} = \frac{b_{32} W_{o2}^2}{-b_{31} b_{42} + b_{32} b_{41}}$$

$$g_{21} = \frac{b_{41} W_{o1}^2}{-b_{31} b_{42} + b_{32} b_{41}}; \quad g_{22} = \frac{b_{31} W_{o2}^2}{-b_{31} b_{42} + b_{32} b_{41}}$$

Full implementation of the technique LMIs-Sliding mode control, monitoring indicates a reference model for both Theta1 (see Fig. 8) to Theta2 (see Fig. 9).

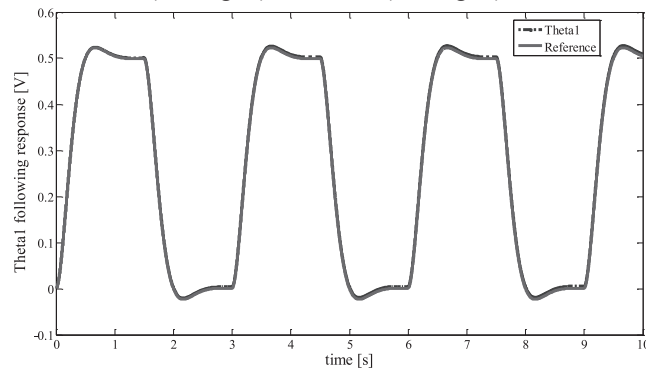


Fig. 8 Theta1 following response LMI-based sliding mode.

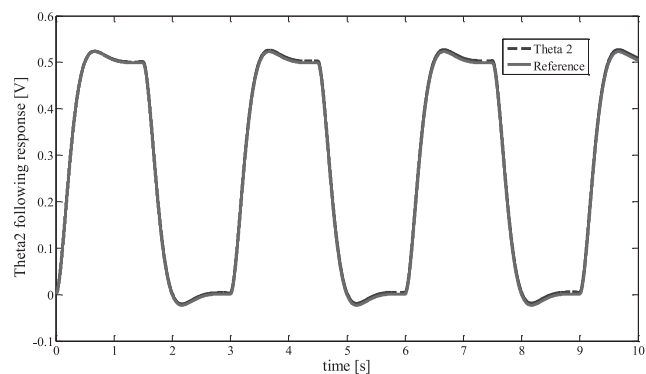


Fig. 9 Theta2 following response LMI-based sliding mode.

In Fig. 10 shows the peaks in the speed and Theta2 Theta1, when a change of the reference model.

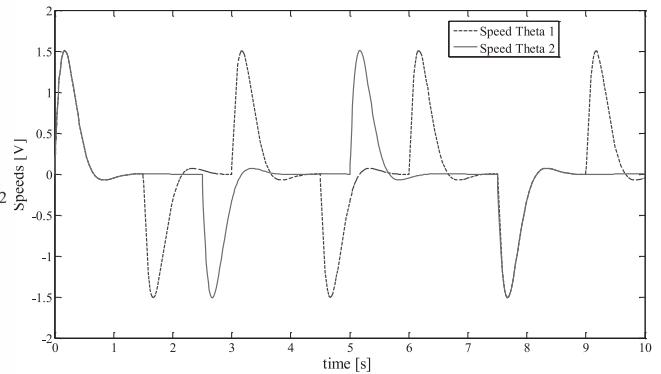


Fig. 10 Speed signals concerning the control-based sliding mode LMIs.

Fig. 11 shows the signals generated by applying the technique surface-Sliding mode control LMIs.

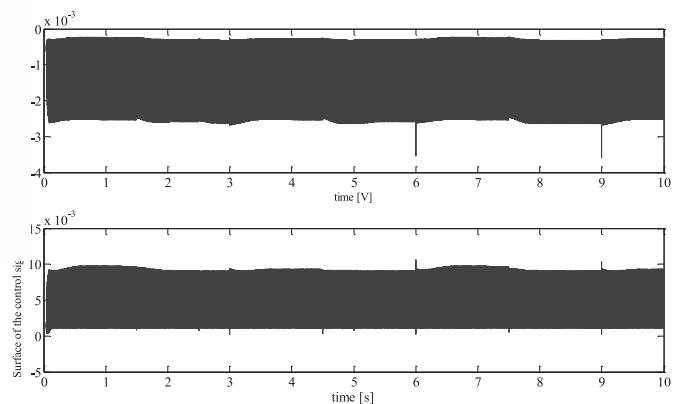


Fig. 11 Surface of the control signals based on LMIs-sliding modes.

It can be seen in Figure 12 the control signals, shows the chattering, according to the signal generated surface.

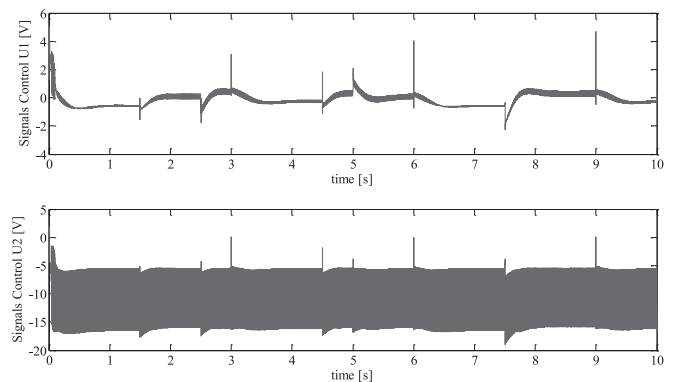


Fig. 12 Control signals concerning the control-based sliding mode LMIs.

VIII. CONCLUSIONS

The control law proposed allows controlling the system in despite uncertainties and internal and external disturbances. The control law is composed by two components; each one is designed in separated form: the linear control component is designed to track the reference model in absence of uncertainties and disturbances; the nonlinear control component assures $X_2(t)$ converges to zero: The velocity and position errors converge to zero.

The control law assures the stability and the model tracking error tends to zero. Also it observes the robustness due a linear component and a nonlinear control component; the robustness of the control law is achieved by imposing a condition on the matrix P and by a proper selection of the sliding mode surface.

The control law was tested in two examples of robotics: the *PR* and the *ELBOW* manipulators and the performance in both cases was satisfactory.

REFERENCES

- [1] J. Craig, *Robótica*. Pearson Education, 2006, pp. 173–188.
- [2] O. I. Higuera-Martínez and J. M. Salamanca, “Diseño de un controlador LMI+I basado en optimizaron para un helicóptero de dos grados de libertad,” *Ing. Investig. y Desarro.*, vol. 5, pp. 22–28, 2007.
- [3] J. M. Salamanca, E. J. Avella-Rodríguez, and R. A. Plazas-Rosas, “Control por LMI-Modos deslizantes aplicado a una clase de sistemas no lineales,” in *VII Congreso Internacional sobre Innovación y Desarrollo Tecnológico. CIINDET 2010*, 2010, pp. 1–8.
- [4] S. Zein-Sabatto, J. Sztipanovitis, and G. E. Cook, “A multiple-control strategy for robot manipulators,” in *IEEE Proceedings of the SOUTHEASTCON '91*, pp. 309–313.
- [5] I. Bonilla, E. J. González-Galván, C. Chávez-Olivares, M. Mendoza, A. Loreda-Flores, and F. Reyes, “A vision-based, impedance control strategy for industrial robot manipulators,” in *2010 IEEE International Conference on Automation Science and Engineering*, 2010.
- [6] S. P. Chan and W.-B. Gao, “Variable structure model-reaching control strategy for robot manipulators,” in *Proceedings, 1989 International Conference on Robotics and Automation*, pp. 1504–1508.
- [7] S. Islam and X. P. Liu, “Robust Sliding Mode Control for Robot Manipulators,” *IEEE Trans. Ind. Electron.*, vol. 58, no. 6, pp. 2444–2453, Jun. 2011.
- [8] A. H. Al-Bayati, “A robust new nonlinear fault tolerant control for robot via an observer,” in *2011 9th World Congress on Intelligent Control and Automation*, 2011, pp. 529–534.
- [9] B. K. Post and W. J. Book, “A robust nonlinear observation strategy for the control of flexible manipulators,” in *2011 IEEE International Conference on Robotics and Automation*, 2011, pp. 2166–2171.
- [10] J. M. Salamanca and E. E. Rosero Garcia, “Sliding mode control with linear matrix inequalities using only output information,” in *2005 International Conference on Industrial Electronics and Control Applications*, pp. 1–6.
- [11] S. B. and L. E. G. and F. and V. B. E., *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM, 1994.
- [12] O. Pérez and W. Colmenares, “Desigualdades lineales matriciales en el diseño integrado de procesos,” *Jornadas de Automática*, pp. 1–7, 2002.
- [13] J. M. Salamanca and J. M. Ramirez Scarpetta, “Diseño de PSS Utilizando Desigualdades Matriciales Lineales,” in *Memorias del Simposio Internacional sobre la Calidad de la Energía Eléctrica - SICEL*, 2005.
- [14] S. Almeida and R. E. Araujo, “Fault-tolerant control using sliding

mode techniques applied to multi-motor electric vehicle,” in *IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society*, 2013, pp. 3530–3535.

- [15] E. J. Avella and R. A. Plazas, “Design of a Gain Scheduling Control Based on Linear Matrix Inequalities for a Dynamic System,” *IEEE Latin America Transactions*, vol. 10, no. 2. pp. 1459–1465, 2012.
- [16] K. J. Aström and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 1997, pp. 1–557.
- [17] K. H. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River: Prentice Hall, 1996, pp. 3–6.
- [18] J. M. Salamanca and R.-D. OO, “Análisis y diseño de sistemas de control de nivel para dos tanques interconectados aplicando desigualdades matriciales lineales (LMI’s),” *Ing. Investig. y Desarro.*, vol. 3, pp. 61–65, 2006.
- [19] L. Vandenberghe and V. Balakrishnan, “Algorithms and software for LMI problems in control,” *IEEE Control Syst. Mag.*, vol. 17, no. 5, pp. 89–95, 1997.
- [20] J.-J. E. Slotine and W. LI, *Applied Nonlinear Control*. Prentice Hall, 1991.
- [21] E. J. Avella-Rodríguez and R. A. Plazas-Rosas, “Laboratorio Virtual para simular un robot de dos articulaciones ELBOW,” Universidad Pedagógica y Tecnológica de Colombia, 2010.



Juan Mauricio Salamanca was born in Sogamoso, Colombia. Electronic Engineering from Universidad Distrital Francisco José De Caldas, Bogota, Ms. In Automatización Industrial from Universidad Nacional de Colombia, Bogotá and the Engineering Doctor degree from *Universidad del Valle*, Cali, Colombia. He is working as full time professor in the UPTC. His current research interests are technology control applications, mine ventilation control and education in control.



Ramiro Alejandro Plazas Rosas was born in Sogamoso, Colombia. He received the B.S. in Electronics Engineering from UPTC and the Master degree in Automatic from *Universidad del Valle*. His current research interests are technology control applications and education in control.



Edna Joydeth Avella Rodríguez was born in Sogamoso, Colombia. He received the B.S. in Electronics Engineering from UPTC, is currently student the master degree in Automatic from *Universidad del Valle*. His current research interests are technology control applications and biological control systems.