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Quantum computing: from qubits to qudits[1]

Computación cuántica: de qubits a qudits

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***Abstract*—Quantum computing has traditionally relied on qubits, which are two-level quantum systems, but the growing demand for scalability and expressivity has driven interest in qudits, their natural generalization to higher-dimensional systems. By expanding the local Hilbert space to $\mathbb{C}^d$, qudits enable denser information encoding, richer entanglement structures, and more compact representations of multiqubit circuits. This brief review contrasts qubits and qudits, surveys leading physical implementations, and examines representative applications including quantum machine learning, quantum simulation, quantum error correction, quantum communication, and quantum sensing. We highlight algorithmic advantages and practical achievements. Finally, we discuss the key challenges that currently limit the scalability of qudit platforms and outline future research directions, including hybrid qubit–qudit architectures, AI-assisted control, and advances toward fault-tolerant high-dimensional quantum computing.**

***Keywords*—Quantum computing, qudits, high-dimensional quantum systems, Hilbert space.**

***Resumen*— La computación cuántica ha dependido tradicionalmente de los qubits, que son sistemas cuánticos de dos niveles, pero la creciente demanda de escalabilidad y expresividad ha impulsado interés en los qudits, su generalización natural a sistemas de mayor dimensión. Al expandir el espacio de Hilbert local a $\mathbb{C}^d$, los qudits permiten una codificación de información más densa, estructuras de entrelazamiento más ricas, y representaciones más compactas de circuitos multiqubit. Esta breve revisión contrasta qubits y qudits, examina las principales implementaciones físicas, y analiza aplicaciones representativas como machine learning cuántico, simulación cuántica, corrección de errores cuántica, comunicación cuántica y detección cuántica. Se destacan las ventajas algorítmicas y los logros experimentales alcanzados. Finalmente, se analizan los principales desafíos que actualmente limitan la escalabilidad de las plataformas basadas en qudits y se esbozan futuras líneas de investigación, incluyendo arquitecturas híbridas qubit–qudit, control asistido por IA y avances hacia una computación cuántica tolerante a fallos en dimensiones superiores.**

***Palabras clave*—Computación cuántica, qudits, sistemas cuánticos de alta dimensión, espacio de Hilbert.**

I. INTRODUCTION

Quantum computing has traditionally been built upon the concept of the qubit, a two-level quantum system whose state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ encodes information in a superposition of $|0\rangle$ and $|1\rangle$. Over the past few decades, progress in qubit-based platforms has enabled both foundational proof-of-principle experiments and near-term demonstrations of quantum advantage [1], [2], [3], [4], [5], [6], [7]. However, qudit technology — where a qudit is a d-level quantum system with $d > 2$— is emerging as an alternative to qubit-based architectures for quantum computation beyond qubits.

Many physical platforms naturally offer multiple accessible energy levels. By leveraging these additional states, one can extend the computational capabilities of such platforms to qudits. Among the key benefits, qudits offer a larger state space for storing and processing information and enable various control operations to be performed at the same time. Consequently, using qudits can reduce the complexity of experimental setups and quantum circuits, decrease the number of physical carriers required for a given register size, and lead to improved algorithmic efficiency [8].

Pioneering work on high-dimensional quantum systems includes early analyses of entanglement structure in qutrits [9] and foundational studies of nonlocality showing that entangled systems in higher dimensions exhibit stronger violations of local realism than qubits [10], [11]. One of the first explicit high-dimensional quantum cryptography protocols demonstrated that d-level states can enhance security compared to 2-level carriers [12]. Moreover, the development

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of quantum error-correcting codes showed how finite-dimensional qudits can be embedded in harmonic oscillators to protect against small displacement errors in position and momentum, leading to robust storage and fault-tolerant processing of quantum information [13]. These results highlighted the richer structure and enhanced capabilities of high-dimensional Hilbert spaces, motivating later advances in qudit-based quantum technologies.

In this brief review, we provide a concise introduction to qudit-based quantum computing, highlighting the fundamental motivations for moving beyond two-level systems and summarizing representative advances across physical implementations, algorithms, and applications. We give a clear sense of how higher-dimensional systems expand the available state space, enable alternative computational strategies, and offer practical benefits in certain platforms. We also discuss open challenges and future directions that could enable scalable qudit-based quantum processors.

II. From qubits to qudits

A. State space.

The state space is the Hilbert space that defines a quantum system's possible states. For qubits, it is $\mathbb{C}^2$ with basis $\{|0\rangle, |1\rangle\}$. An n-qubits register spans $\mathbb{C}^{2^\wedge n}$, supporting 2^n basis states. This enables quantum parallelism and exponential scaling but requires many physical carriers to implement complex algorithms. Conversely, qudits operate in $\mathbb{C}^d$ with basis $\{|0\rangle, \ldots, |d\text{-}1\rangle\}$, and a general state is described as:

$$|\psi\rangle = \sum_{j=0}^{d-1} \alpha_j |j\rangle, \quad \sum_{j=0}^{d-1} \left|\alpha_j\right|^2 = 1 \tag{1}$$

An n-qudit register spans $\mathbb{C}^{d^\wedge n}$, offering a larger state space per physical system. For instance, a qutrit (d = 3) spans $\mathbb{C}^3$, and two qutrits span $\mathbb{C}^9$, reducing the number of required particles and gates in quantum computations [14].

B. Logic gates.

The following description is based on the framework presented in [15]. In the standard qubit model, quantum circuits operate on two-level systems with native gates that include single-qubit rotations $R_\phi(\theta) = \exp(-i\sigma_\phi\, \theta/2)$ where $\sigma_\phi = \sigma_X \cos(\phi) + \sigma_Y \sin(\phi)$ and entangling operations such as CNOT and controlled-Z (CZ) defined respectively as CNOT: $|x, y\rangle \rightarrow |x, y \oplus x\rangle$ and CZ: $|x, y\rangle \rightarrow (-1)^{xy} |x, y\rangle$ for $x, y \in \{0, 1\}$.

Up to an overall phase factor, any two-dimensional unitary matrix can be written as [16].

$$Y_2(\lambda, \nu, \phi) = \begin{pmatrix} \cos\lambda & -e^{i\nu}\sin\lambda \\ e^{i(\phi-\nu)}\sin\lambda & e^{i\phi}\cos\lambda \end{pmatrix} \tag{2}$$

expressed in the basis states of a qubit, $|0\rangle$ and $|1\rangle$. Multiqubit gates, such as the Toffoli gate CCX = C^2X, defined as C^2X: $|x_1, x_2, y\rangle \rightarrow |x_1, x_2, y \oplus x_1x_2\rangle$, are decomposed into sequences of CNOTs, often requiring ancillary qubits and O(L) entangling gates for L-controlled Toffoli gates.

For qudits, the state space can be decomposed as $\mathbb{C}^d = \mathbb{C}^2 \oplus \mathbb{C}^{d-2}$ for qubit storage in the first two levels and ancillary levels for gate simplification. Native operations include single-qudit rotations $R^\phi_{ij}(\theta) = \exp(-i\sigma^\phi_{ij}\, \theta/2)$, where $\sigma^\phi_{ij} = e^{-i\phi}|i\rangle\langle j| + e^{i\phi}|j\rangle\langle i|$, phase gates $Ph^i(\theta) : |x\rangle \rightarrow e^{i\theta}|x\rangle$ if x=i (and $|x\rangle$ otherwise), and entangling gates such as controlled-phase $CPh^{i|j}$,

$$CPh^{i|j} : \begin{cases} |i,j\rangle \mapsto -|i,j\rangle, \\ |x,y\rangle \mapsto |x,y\rangle \text{ if } (x,y) \neq (i,j) \end{cases} \tag{3}$$

iSWAP,

$$iSWAP^{ij|kl}(\theta) : \begin{cases} |i,k\rangle \mapsto e^{i\theta}|j,l\rangle, \\ |j,l\rangle \mapsto e^{i\theta}|i,k\rangle, \end{cases} \tag{4}$$

or the Mølmer–Sørensen XX gate.

$$XX^{ij|kl}_{\varphi,\theta}(\chi) = \exp(\text{-}i\sigma^{ij}_\varphi \otimes \sigma^{kl}_\theta\, \chi) \tag{5}$$

Finally, the qubit Pauli group, spanned by the operators σ_X and σ_Z is generalized in d dimensions by the shift and clock operators:

$$\sigma^{(d)}_X : |x\rangle \mapsto |x + 1 \bmod d\rangle, \quad \sigma^{(d)}_z : |x\rangle \mapsto e^{i2\pi x/d}|x\rangle \tag{6}$$

C. Information capacity.

A qudit encodes $\log_2(d)$ bits of classical information, leading to a higher information density than qubits, which encode one bit. For example, a qutrit stores approximately 1.58 bits, and a ququart (d = 4) stores 2 bits, meaning that two qutrits already encode more information than two qubits. This implies that representing a Hilbert space of dimension N, which requires $n_2 = \log_2 N$ qubits, can be achieved with $n = n_2 / \log_2 d$ qudits. Thus, multivalued quantum systems provide a logarithmic reduction in the number of physical carriers needed to span the same quantum memory. Additionally, multivalued architectures can offer a $(\log_2 d)^2$ improvement in time complexity of a multivalued simulation compared to binary constructions [16].

III. Experimental platforms

A. Trapped ions.

Trapped ions' rich internal structure enables the encoding of qudits, allowing a single ion to serve as a multilevel quantum system [17]. Information is typically stored in the ions' metastable states, which can be manipulated using optical or microwave fields to execute quantum operations [18]. This platform has already demonstrated universal qudit quantum computation [19] through the realization of multiqubit gates using single-qudit operations and the Mølmer–Sørensen gate, showing how additional levels can be utilized for the decomposition of multiqubit gates [20], and by the implementation of a universal gate set for qudit quantum information processing with performance close to that of qubit systems while requiring minimal overhead in classical control capabilities [21].

More recently, a native two-qudit gate (up to d = 5) in a trapped-ion setup successfully generated genuine high-dimensional entanglement without a scaling overhead in calibration as d increased and through just a single application of the gate [22]. Meanwhile, the first results on manipulating multiqudit systems based on $^{171}Yb^{+}$ ions with optical qudits realized a two-ququart quantum processor with single-qudit gates fidelities ranging from 83% to 89% [18].

B. Photonic systems.

Photonic systems offer various quantum properties that can be used to represent qudit states. Traditionally, spontaneous parametric down-conversion (SPDC) has been used to generate photonic quantum states entangled in several degrees of freedom, such as time, polarization, or energy. Specifically, high-dimensional entangled qudits generated using orbital angular momentum (OAM) correlations represent a powerful implementation that allows a single carrier to transport large amounts of information, given the unbounded nature of OAM [23], [24]. Elements such as cylindrical lenses or spiral phase plates can reshape the wavefront of an initial Gaussian beam to create OAM-encoded qudits. A nonzero OAM is characterized by a phase factor $e^{il\theta}$, where l represents the quantum number that quantifies the OAM. Since it can take any integer value without an upper limit, this encoding enables the representation of an arbitrarily large Hilbert space. Additionally, time and frequency provide alternative methods. Time-bin encoding takes advantage of the time at which a photon is emitted as a degree of freedom. By using a pulsed laser, each photon can be associated with a specific time slot, forming a discrete basis of time states. Frequency-encoded qudits, on the other hand, can be obtained by pumping a microring resonator to generate pairs of photons in a superposition of multiple frequency modes [25].

C. Superconducting circuits.

Superconducting transmon qubits inherently exhibit an anharmonic ladder of energy levels due to the nonlinearity of their Josephson inductance, making the oscillator suitable for multilevel operations through selective addressing of individual energy transitions. Coherent control of higher excited states has been demonstrated by applying sequential π-pulses to drive transitions from the ground state. Measurements reveal beating patterns in Ramsey fringes that provide a direct measurement of charge dispersion at higher levels as well as lifetimes exceeding 20 µs for states up to 4, confirming the suitability of multilevel systems for quantum information applications and simulations [26].

Recent progress has explored a protocol based on parametric coupling in which partial state swaps enable a high-fidelity controlled-Z gate between two fixed-frequency transmon qutrits by modulating a flux-tunable coupler. This approach offers an advantage in terms of full control and the avoidance of infidelity sources associated with tunable components [27]. Advances in error characterization and effective mitigation have further expanded the computational viability of superconducting qudits. Notably, tailored suppression of Markovian noise has been demonstrated on a multiqutrit transmon, achieving up to a threefold improvement in multipartite qutrit entanglement and random-circuit sampling performance. This ability to execute substantially deeper circuits with minimal error accumulation is particularly critical for fully exploiting the advantages of qudits in quantum algorithms and high-fidelity quantum simulation [28].

IV. Quantum algorithms

Quantum algorithms are usually formulated in the qubit circuit model, where computation is implemented by sequences of one- or two-qubit gates acting on two-level registers [29]. In the case of qudits, this scheme is naturally generalized to systems of dimension d, so that the elementary operations belong to U(d) and Pauli-type operators are replaced by shift and clock operators that generate the generalized Pauli group, which is closely related to the Clifford group and Clifford algebras [8], [15]. On this basis, quantum algorithms such as Deutsch–Jozsa, Bernstein–Vazirani, Grover, or Shor can be described as particular cases of a more general framework, in which the local dimension is no longer restricted to two levels [20].

The multivalued gate model considers registers of qudits in which each carrier has d levels {|0⟩, …, |d-1⟩, and elementary gates capable of performing unitary transformations between these levels. Extensions of binary quantum logic, such as the multivalued gates of Muthukrishnan–Stroud [16], provide sets of gates for computation with qudits and make it possible to decompose any operation in $\mathbb{C}^d$ into sequences of rotations and multilevel controlled gates [14]. This framework preserves the conceptual structure of the best-known quantum algorithms, for example, the Hadamard gates that prepare uniform superpositions in algorithms such as Deutsch–Jozsa or Bernstein–Vazirani are generalized to Fourier transforms in base d, while the controlled phases that implement oracles and arithmetic modules in Grover or Shor can be realized by multivalued phase gates acting on a reduced number of carriers. The fundamental difference between the use of qubits and qudits, therefore, lies in the required physical resources (number of carriers, circuit depth, number of entangling gates) and in the control complexity that is transferred to each elementary gate [15].

A direct consequence of working with carriers of dimension d is that the unused levels of the logical subspace can be employed as auxiliary workspace. In practice, a common strategy is to embed logical qubits in the first two levels of each qudit and use the remaining levels as a transient workspace. In this scheme, the states |0⟩ and |1⟩ encode the logical information, while the states |2⟩, …, |d-1⟩ are temporarily occupied during the implementation of multiqubit gates and are emptied at the end of the circuit. This idea makes it possible to design more compact decompositions of highly controlled gates, reducing the number of entangling gates and the need for additional physical ancillas, without modifying the logical outcome of the algorithm [30], [31].

From this perspective, reference algorithms such as Deutsch–Jozsa and Bernstein–Vazirani provide minimal examples in which the use of extra levels simplifies the implementation of the oracle: the balanced or constant function can be encoded as a single multilevel phase gate instead of a network of binary controlled gates. Similarly, the oracle and reflection blocks of Grover, which depend on multi-controlled phase gates, can be compressed using auxiliary levels, which reduces the circuit depth [15], [30]. In more complex algorithms, such as Shor, the modular arithmetic modules (addition, multiplication and phase control) also benefit from the availability of additional levels, since they allow more efficient decompositions of multi-controlled gates in the enlarged Hilbert space. In operational terms, the information can be processed in quantum digits instead of bits, which leads to more compact implementations of arithmetic operations on registers of fixed size [32].

The use of qudits, however, introduces new trade-offs in the resource balance. On the one hand, a larger local dimension makes it possible to represent the same problem size with fewer carriers and to design circuits with a smaller number of entangling gates and reduced logical depth [33]. On the other hand, each elementary gate requires finer coherent control over multiple transitions, which tends to increase the complexity of the associated classical control, as well as the susceptibility to leakage errors and spurious couplings between levels [34]. In the regime of noisy intermediate-scale quantum (NISQ) devices, qudit-based algorithms essentially propose a trade-off: accepting greater complexity in control at the individual level in exchange for reducing the total number of carriers and the number of entangling gates in the most costly algorithmic sub-modules [35].

V. Applications

A. Quantum Machine Learning.

One of the most promising applications of quantum computing is Quantum Machine Learning (QML), which has the potential to solve certain problems faster than classical computing while also reducing energy consumption and environmental impact [36]. The extension of qubit-based quantum algorithms to qudits has been explored, as this alternative could offer advantages in terms of expressiveness and computational efficiency. For example, an analysis of qudit machine learning was conducted by applying variational algorithms to real datasets, and results indicate that when the qudit dimension approaches the number of data features, performance improves compared to classical models [36]. Variational Quantum Circuits (VQCs) and Quantum Neural Networks (QNNs) have also been investigated, employing qutrits for binary and multiclass classification tasks. The accuracy of a QNN with qutrits against a VQC with qubits and a classical Support Vector Machine (SVM) was compared; although the SVM model outperformed the quantum approaches, the qutrit circuit exhibited better performance than the VQC with qubits [37].

Regarding the adaptation of QNNs to qudits, a comparative analysis between qubits and qutrits using Variational Quantum Neural Networks (VQNNs) was performed, and it was found that, across all evaluated datasets, the qutrit-based approach outperformed the qubit models when using an amplitude encoding scheme [38]. Meanwhile, a qudit-based QNN using the Quantum Fully Self-Supervised Neural Network (QFS-Net) architecture for automated MRI brain image segmentation achieved accuracy rates above 99% with minimal computational resource usage [39]. Another qutrit-based classification model achieved 90% accuracy using only 10% of a total of 800 data points from the Iris dataset [40]. When compared to a VQC based on four qubits, the latter achieved better accuracy, but the classifier based on a single qudit demonstrated the capability to perform multiclass classification on multidimensional data.

Other methods employed in the context of QML include data re-uploading [41], Density Matrix Kernel Density Classification (DMKDC) [42], and the Tree-like Tensor Networks (TTNs) [43]. These models successfully learned from various datasets and achieved performance comparable to classical machine learning models.

B. Quantum simulation.

Quantum simulation plays a crucial role in understanding complex quantum systems and materials, and qudits can offer significant advantages in advanced measurements and observable extraction. As reported in [44], leveraging the higher-dimensional Hilbert space of a qudit to implement IC-POVMs (Informationally Complete Positive Operator-Valued Measures), which are typically realized by coupling additional ancilla qubits to each logical qubit, offers a practical solution to the demanding requirements for device size and connectivity. This approach enables the use of generalized quantum measurements for extracting operator expectation values by performing projective measurements within the additional states provided by the qudit, eliminating the need for ancillary qubits. Additionally, another study implemented nonorthogonal measurements, specifically Symmetric Informationally Complete POVMs on an ion-trap processor without the need for ancillary ions to assist in the measurement process. Utilizing additional energy levels within each ion, the SIC-POVM was locally mapped to orthogonal states embedded in a higher-dimensional system, introducing an alternative to traditional quantum state tomography, as this method requires only a single measurement setting, addressing the computational costs associated with scaling to larger qubit systems [45].

Other advances have highlighted the suitability of qudits for quantum simulation tasks, particularly in lattice gauge theories. On the experimental side, a (1+1)D SU(2) non-Abelian lattice gauge theory was digitally simulated using a six-level trapped-ion qudit processor, demonstrating that multilevel ions can naturally encode gauge degrees of freedom and enable efficient simulation of the model with notably short circuit depth [46].

Proposals with Rydberg-atom qudits have shown that representing discretized gauge fields with qudits can yield higher fidelities and significantly reduce experimental resources compared with qubit-only approaches, given the more natural match between the simulating and the simulated degrees of freedom, which preserves the local structure of gauge-invariant interactions [47]. Additionally, a mapping of fermionic Hamiltonians onto qudit registers was proposed using the auxiliary fermion method, reducing the number of computational units to be controlled, which is particularly beneficial for highly connected target Hamiltonians [48].

C. *Quantum Cryptography and Communication.*

A key advantage of qudits in quantum cryptography and communication is their higher robustness against noise. Since the security of most quantum protocols relies on keeping channel error rates below a certain threshold, any increase in noise tolerance translates into higher tolerable error rates and, consequently, more practical implementations. It has been shown that this noise resilience grows with the local dimension d of the qudit. Furthermore, qudits strengthen the security of quantum communication protocols by reducing the fidelity of any possible cloning attack, and they may enable longer secure transmission distances [25].

A pioneering field test of single-photon high-dimensional Quantum Key Distribution (QKD) was performed over a 300 m turbulent free-space link using a four-level BB84 protocol with structured photons, attaining a quantum bit error rate of 11% and a secure key rate of 0.65 bits per sifted photon, demonstrating the feasibility of increasing security in data transmission [49]. A more recent proof-of-principle experiment implemented a time-phase high-dimensional BB84 protocol using a single-photon detector per basis and exploiting the temporal Talbot effect for phase-encoded superposition detection, again supporting increased robustness and secret key rates for the four-dimensional encoding in contrast to the two-dimensional case [50].

Moreover, other studies have reported advances that include support for states with dimensionality up to $d = 15$ in integrated photonic platforms, and stable transmission of qudits in multicore and multimode fibers, as well as in free-space links distributing OAM and hyperentangled photons [25]. These developments, along with other experimental techniques for the generation and manipulation of high-dimensional entanglement proposed in discretized degrees of freedom of photons, underscore the suitability of high-dimensional encodings, where qudits can enhance tasks such as teleportation and, potentially, entanglement swapping [51].

D. *Quantum Error Correction.*

Quantum error correction (QEC) protects quantum information from decoherence and operational errors by redundantly encoding logical states into stabilizer codes. Increasing the local dimension can notably enhance the performance of fault-tolerant protocols. In particular, a quantum Reed–Muller code in the qudit setting with transversal non-Clifford gates used $n = d - 1$ qudits and detected on the order of d/3 errors, exhibiting improved behavior for magic-state distillation (MSD) with both threshold and efficiency increasing systematically with the dimension d. This stands in contrast to qudit toric codes, in which enhanced efficiency does not occur [52].

Simulations of the smallest qudit error-correction code under circuit-level noise showed that, despite noise models whose strength scales with dimension, qudits of $d = 3$ and $d = 5$ can achieve error thresholds (on the order of 10^{-4}) comparable to qubits when equipped with adapted decoders and additional flag qudits to suppress hook errors [53]. Complementarily, recent theoretical work has established a general, symmetry-driven framework for constructing error-correcting codes for qudits using irreducible representations of SU(d), reducing the Knill–Laflamme conditions to only three constraints and enabling an infinite family of codes that encode a logical qudit into $(d-1)^2$ physical qudits [54]. In addition, experimental demonstrations have now realized error-corrected logical qutrits and ququarts using GKP bosonic encodings, achieving beyond break-even performance with gains exceeding 1.8. Moreover, a way for concatenating codes internally was highlighted, as various layers of error correction could be implemented inside just one oscillator by embedding a logical qubit within a bosonic logical qudit [55].

E. *Quantum Sensing and Metrology.*

Qudit-based platforms constitute powerful tools for high-precision quantum sensing and metrology as well, offering performance enhancements in resolution and sensitivity across sensing architectures. A key example is the generalization of Ramsey interferometry to qudits in Wigner–Majorana systems across dimensions 2 to 7, where results showed that the number of Ramsey fringes increases linearly with d for fixed interrogation time, enabling substantial improvements in spectral resolution. Qutrits achieved a twofold increase in resolution without contrast degradation, while ququints approached a fourfold increase, positioning single-qudit interferometry as a practical route to high-precision quantum metrology and sensing technologies without requiring additional experimental resources or entanglement overhead [56].

Superconducting multilevel systems offer another concrete instance of qudit-enabled metrological enhancement. In transmon devices operated as qutrits, the use of a base-3 semi-quantum Fourier transform enabled magnetic-field sensing with a theoretical precision improvement by a factor of two compared to the qubit mode [57]. Beyond this gain, the qutrit protocol also reduced the number of required Fourier-transform iterations by a factor of approximately 0.63, providing a practical improvement in resource efficiency. Finally, multilevel advantages also extend to fundamental physics searches. A universal framework for qutrit-based sensing has been shown to yield a sequence-independent fourfold increase in quantum Fisher information (QFI) and a twofold gain in projection-noise-limited sensitivity [58]. Applied to ultralight dark-matter searches using NV-center qutrits, this leads to an improvement of up to an order of magnitude in the reach for axion–electron couplings relative to qubit protocols. Since the improvement follows from general spin-algebra properties rather than NV-specific physics, the same principle applies to superconducting, trapped-ion, and neutral-atom platforms, establishing multilevel quantum sensing as a broadly applicable tool for precision metrology.

VI. Challenges and Future Perspectives

Despite their advantages, implementing and scaling qudit platforms presents intertwined challenges. Building on the earlier discussion of resource trade-offs in qudit algorithms, these benefits come at the cost of a more demanding operation. Multilevel solid-state systems, in particular, are intrinsically more vulnerable to cross-talk, leakage to unwanted levels, and decoherence, complicating precise control [59]. As the dimensionality d increases, the number of allowed transitions increases as well, expanding the space of possible error pathways. Consequently, the calibration of multilevel gates demands sophisticated pulse-shaping techniques and extensive manual tuning [60].

While theoretical analyses show that multivalued simulations can offer a time advantage over binary circuits, this benefit comes at the cost of physically larger elementary gates that must coherently address all d levels instead of just two [16]. Furthermore, the software ecosystem for qudits remains comparatively underdeveloped, as most existing quantum toolchains are rooted in binary logic, so transitioning to high-dimensional systems requires new abstractions and programming libraries, and many established techniques must be re-engineered for the qudit setting [60]. Additionally, universal gate constructions and characterizations become harder to implement, and error-correction codes become more involved as the Clifford hierarchy grows in complexity for larger d, complicating theoretical analyses and experimental implementations [8].

A natural path forward is to explore hybrid qubit–qudit architectures, in which qudits are used selectively to encode multivalued data, compress ancilla overhead, or implement high-dimensional entanglement, while qubits retain their role in subroutines where binary logic remains advantageous [8]. Concrete schemes along these lines already demonstrate how embedding qubits into qudits or using qudit levels to compress qubit circuits can reduce ancilla requirements and entangling depth in practice [61].

The calibration burden of multilevel systems can also be mitigated, at least in part, by control schemes assisted by artificial intelligence. Recent work on quantum machine learning with qudits and on multilevel processors, together with the development of optimal-control techniques and differentiable programming, indicates that approaches based on reinforcement learning and neural-network controllers could help optimize pulse sequences, identify leakage pathways, and adapt control protocols in real time [59], [60]. As these AI-driven methods mature, they could significantly reduce the amount of manual tuning that currently limits the scalability of high-dimensional platforms in the NISQ regime [62].

Beyond local processors, integrating qudits into quantum networks and distributed architectures offers another promising avenue, since high-dimensional entanglement can increase channel capacity and protocol robustness, and enable more efficient quantum repeaters [8]. In parallel, software frameworks tailored to qudits and mixed-dimensional systems, such as MQT Qudits, will be key to describing and compiling circuits on heterogeneous hardware [63]. In the long term, advances in qudit stabilizer codes and generalized Clifford constructions point toward fault-tolerant quantum computing with $d > 2$, and demonstrating logical qudits together with clear advantages in selected applications will be decisive to justify the additional complexity of high-dimensional architectures [15], [60].

VII. Conclusions

In conclusion, qudits are a competitive alternative to the traditional qubit paradigm, offering a compelling path toward more expressive quantum information processing and enabling richer entanglement structures, more compact encodings, and reductions in circuit depth for a variety of algorithms and applications. Their implementation across different platforms shows that these advantages are not merely theoretical; multilevel systems are already experimentally accessible and actively being explored. However, these benefits come with substantial demands. As dimensionality increases, control becomes more fragile, error mechanisms proliferate, and the software ecosystem remains far less mature than its qubit counterpart. Even so, qudits hold significant promise, and progress in both hardware control and high-dimensional software tools will be essential to move them from specialized demonstrations toward scalable quantum technologies.

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